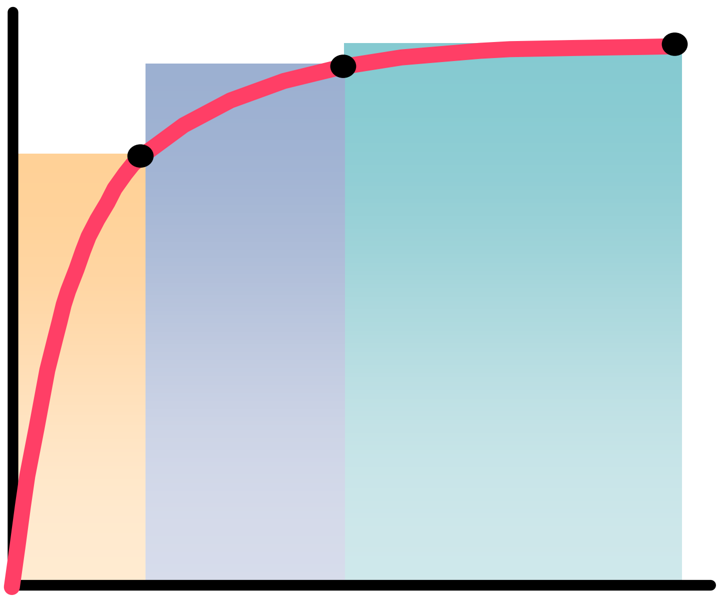
Inventory Management and more

PART IV



FORECASTS

Demand management

- **forecasting**
- **Order processing**

Characteristics

- **Demand patterns to be identified**
- **Trend**
- **Seasonality**
- **Random variation**
- **Stable versus dynamic**
- **Dependent versus independent demand**

FORECASTS

Collection and preparation of data

Forecasting techniques

- Qualitative…on judgment
- Extrinsic : external factors
- Intrinsic
  - Average demand if quite steady
  - Moving averages with little seasonality
  - Exponential smoothing : the new data can be given any weight wanted

FORECASTS

- Seasonality
  - Index
  - Forecasts and annual demand average
  - Deseasonalized demand
  - Forecast error
  - Mean absolute deviation
  - Normal distribution
  - Production lead time / Demand lead time ratio

Patterns of Demand

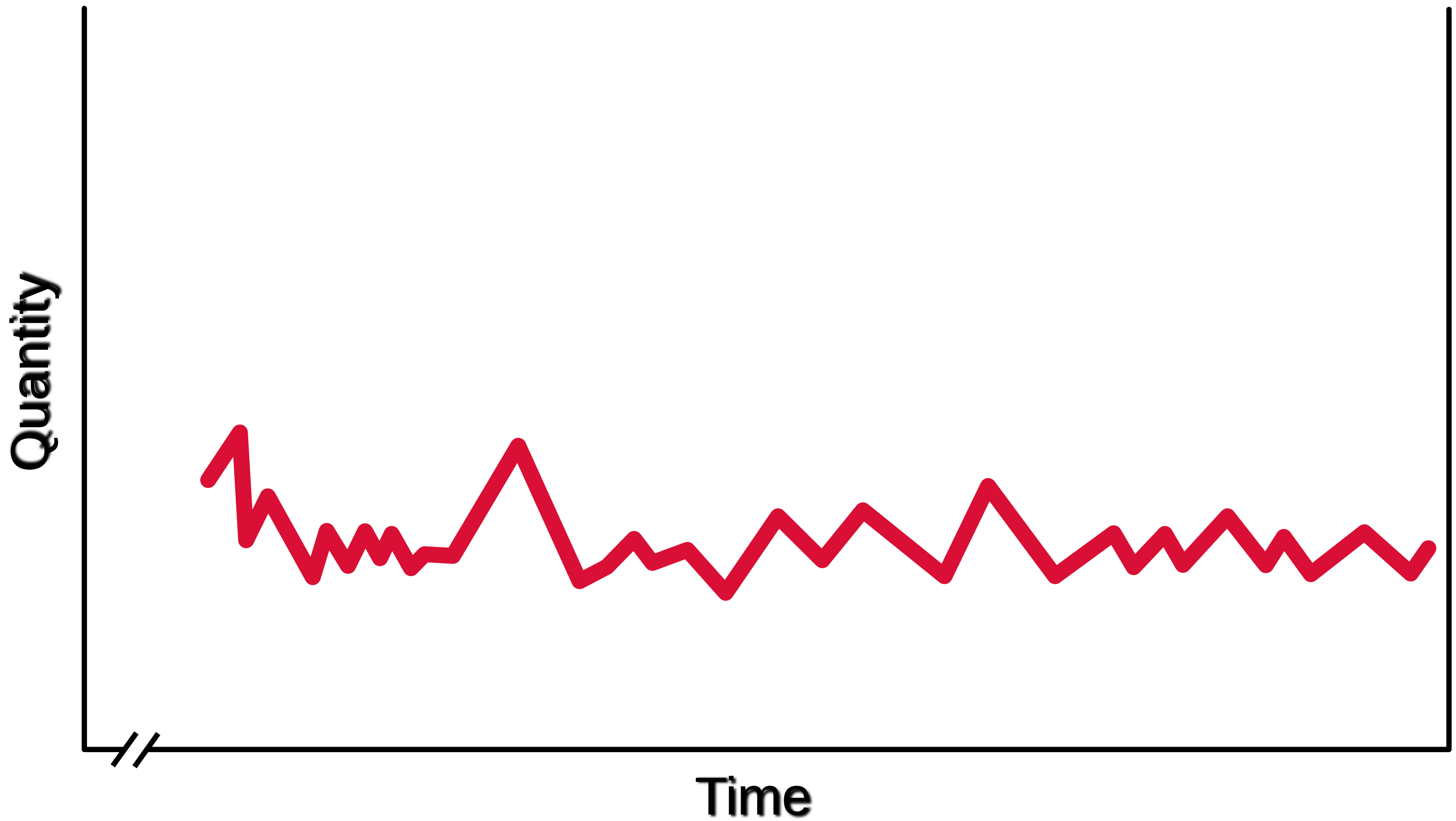
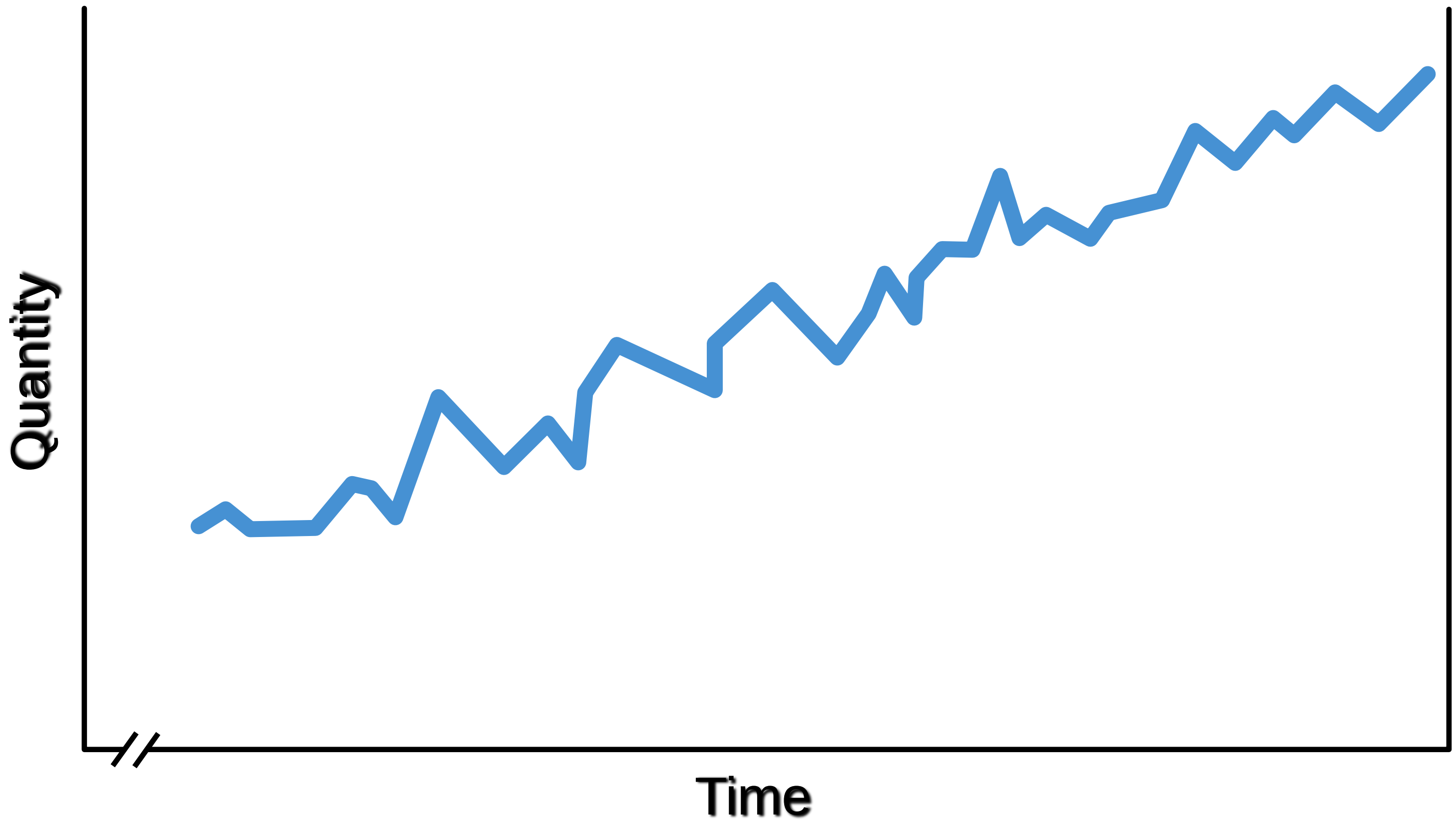


Figure 13.1

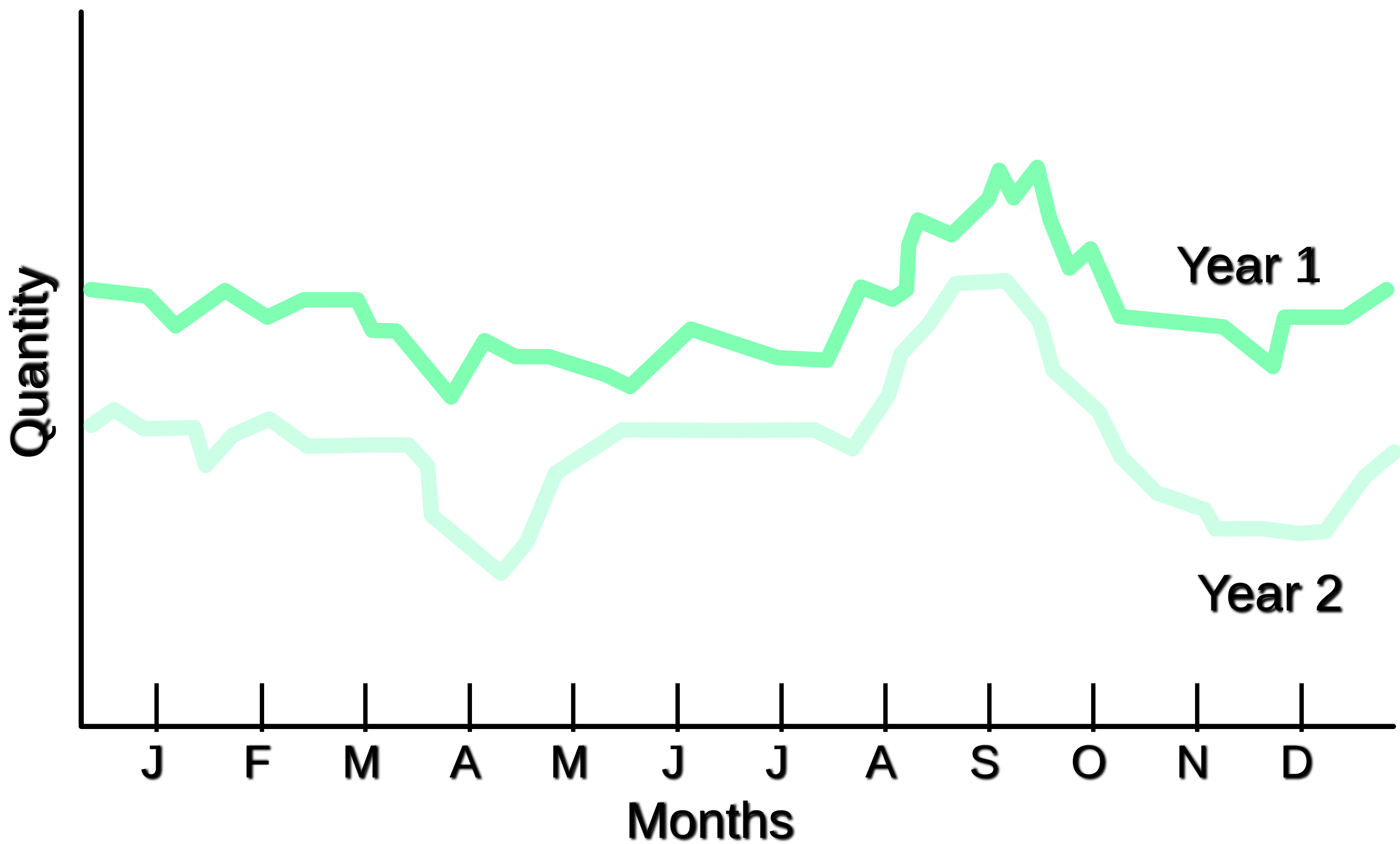
(a) Horizontal: Data cluster about a horizontal line.

Patterns of Demand



(b) Trend: Data consistently increase or decrease.

Patterns of Demand



(c) Seasonal: Data consistently show peaks and valleys.

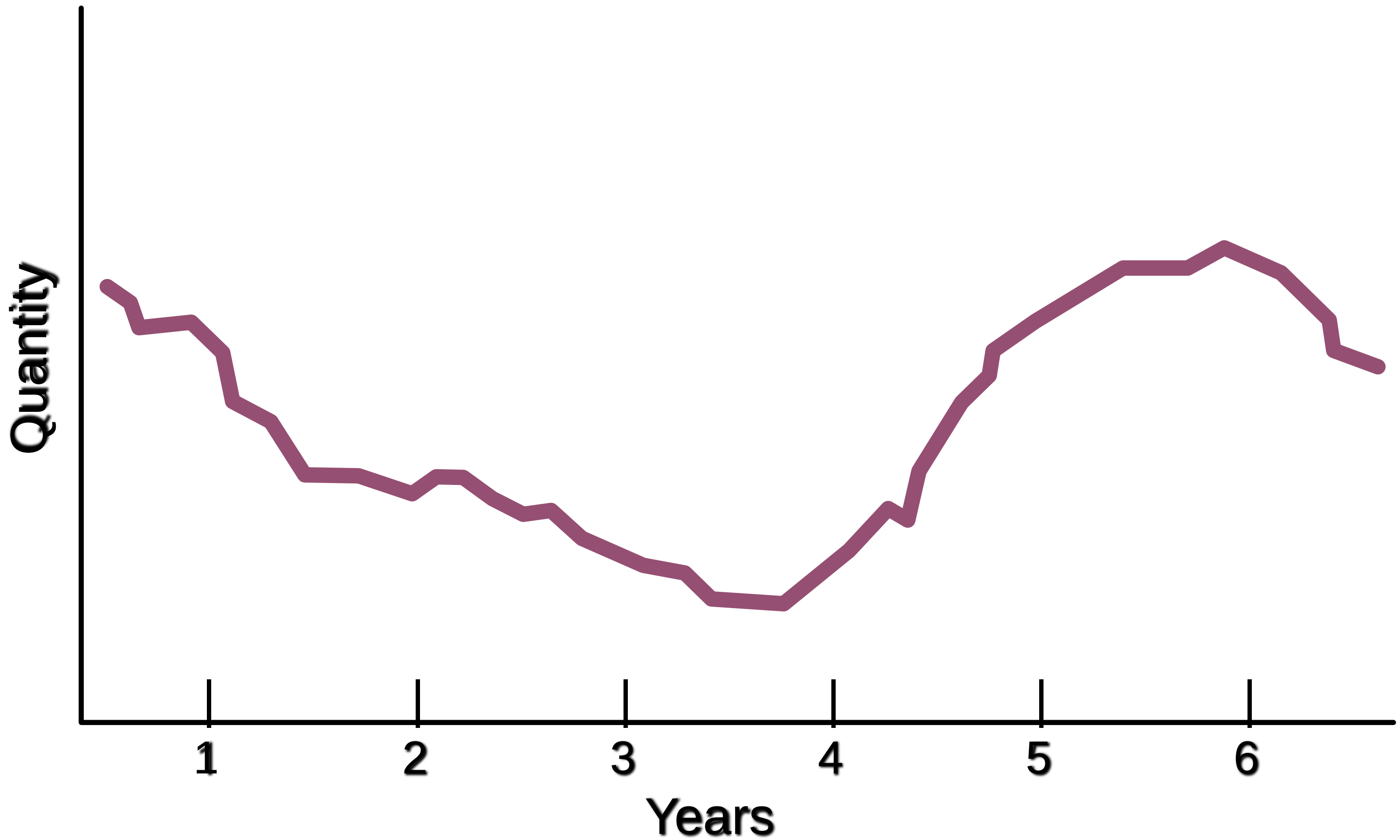
FUTURE TIME HORIZON

- Short-range forecast
  - This forecast has a time span of **up to 1 year** but is generally less than 3 months. It is used for planning purchasing, job scheduling, workforce levels, job assignments, and production levels.

- Medium-range forecast
  - A medium-range, or intermediate, forecast generally spans from 3 months to 3 years. It is useful in sales planning, production planning and budgeting, cash budgeting, and analysis of various operating plans.

- Long-range forecast
  - Generally 3 years or more in time span, long-range forecasts are used in planning for new products, capital expenditures, facility location or expansion, and research and development.

Patterns of Demand



(d) Cyclical: Data reveal gradual increases and decreases over extended periods.

Demand forecast application

Economic forecasts

- ***Planning indicators that are valuable in helping organizations prepare medium- to long-range forecasts.***

Technological forecasts

- ***Long-term forecasts concerned with the rates of technological progress.***

Demand forecasts

- ***Projections of a company's sales for each time period in the planning horizon.***

Connection with techniques

TABLE 13.1 DEMAND FORECAST APPLICATIONS

Application	Time Horizon Short Term (0–3 months)	 Medium Term (3 months–2 years)	 Long Term (more than 2 years)
Forecast quantity	Individual products or services	Total sales Groups or families of products or services	Total sales
Decision area	Inventory management Final assembly scheduling Workforce scheduling Master production scheduling	Staff planning Production planning Master production scheduling Purchasing Distribution	Facility location Capacity planning Process management
Forecasting technique	Time series Causal Judgment	Causal Judgment	Causal Judgment

Qualitative method

Jury of executive opinion

- ***A forecasting technique that uses the opinion of a small group of high-level managers to form a group estimate of demand.***

Delphi method

- ***A forecasting technique using a group process that allows experts to make forecasts.***

Sales force composite

- ***A forecasting technique based on salespersons' estimates of expected sales.***

Market survey

- ***A forecasting method that solicits input from customers or potential customers regarding future purchasing plans.***

FORECASTS

- **Principles**
  - Forecasts are usually wrong
  - Every forecast should include an estimate of error
  - Forecasts are more accurate for families or groups
  - Forecasts are more accurate for nearer time periods

FORECASTS

Collection and preparation of data

Forecasting techniques

- Qualitative…on judgment
- Extrinsic : external factors
- Intrinsic
  - Average demand if quite steady
  - Moving averages with little seasonality
  - Exponential smoothing : the new data can be given any weight wanted

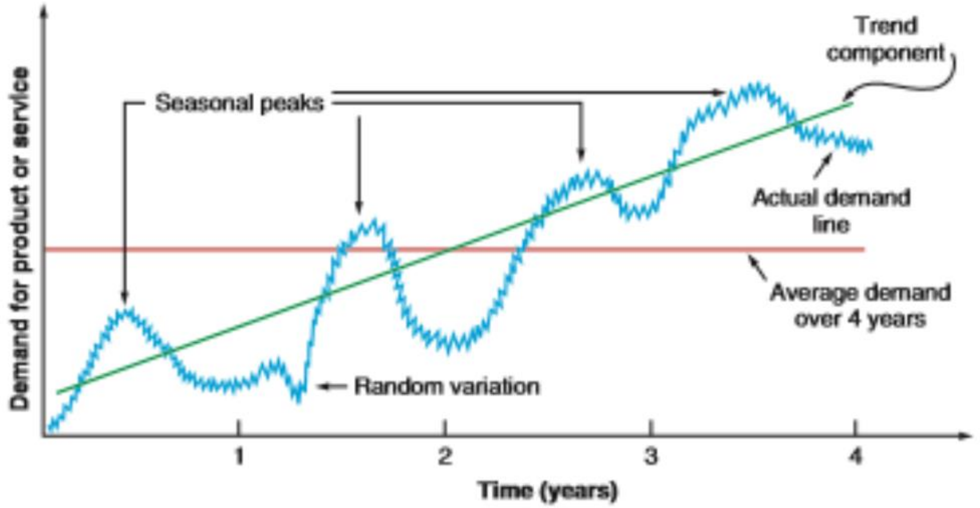
Sum up

ABOUT FORECASTS TRENDS

ABOUT TIME

WHAT KIND OF MEASURES' APPROACH

Naive approach



Moving averages

- A forecasting method that uses an average of the n most recent periods of data to forecast the next period.

Donna's Garden Supply wants a 3-month moving-average forecast, including a forecast for next January, for shed sales.

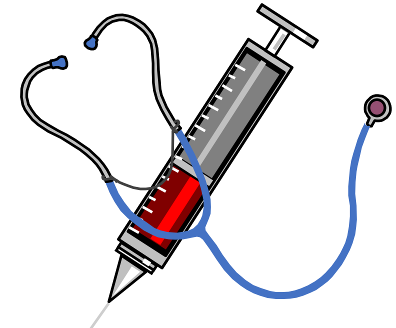
APPROACH ▶ Storage shed sales are shown in the middle column of the following table. A 3-month moving average appears on the right.

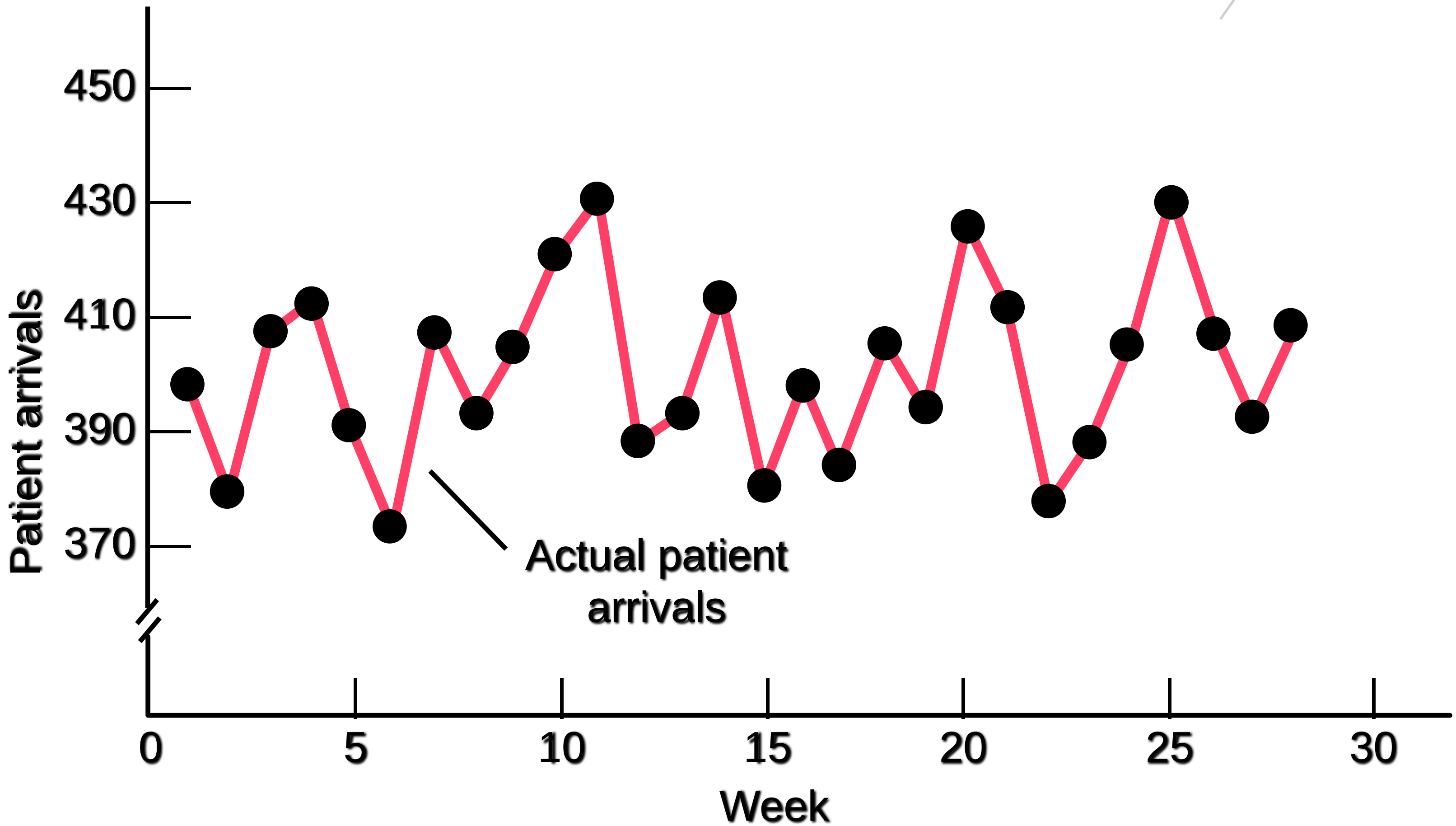
MONTH	ACTUAL SHED SALES	3-MONTH MOVING AVERAGE
January	10	
February	12	
March	13	
April	16	$(10 + 12 + 13)/3 = 11\frac{2}{3}$
May	19	$(12 + 13 + 16)/3 = 13\frac{2}{3}$
June	23	$(13 + 16 + 19)/3 = 16$
July	26	$(16 + 19 + 23)/3 = 19\frac{1}{3}$
August	30	$(19 + 23 + 26)/3 = 22\frac{2}{3}$
September	28	$(23 + 26 + 30)/3 = 26\frac{1}{3}$
October	18	$(26 + 30 + 28)/3 = 28$
November	16	$(30 + 28 + 18)/3 = 25\frac{1}{3}$
December	14	$(28 + 18 + 16)/3 = 20\frac{2}{3}$

$$\text{Moving average} = \frac{\sum \text{demand in previous } n \text{ periods}}{n}$$

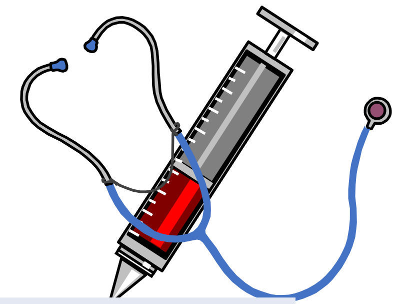
where n is the number of periods in the moving average—for example, 4, 5, or 6 m respectively, for a 4-, 5-, or 6-period moving average.

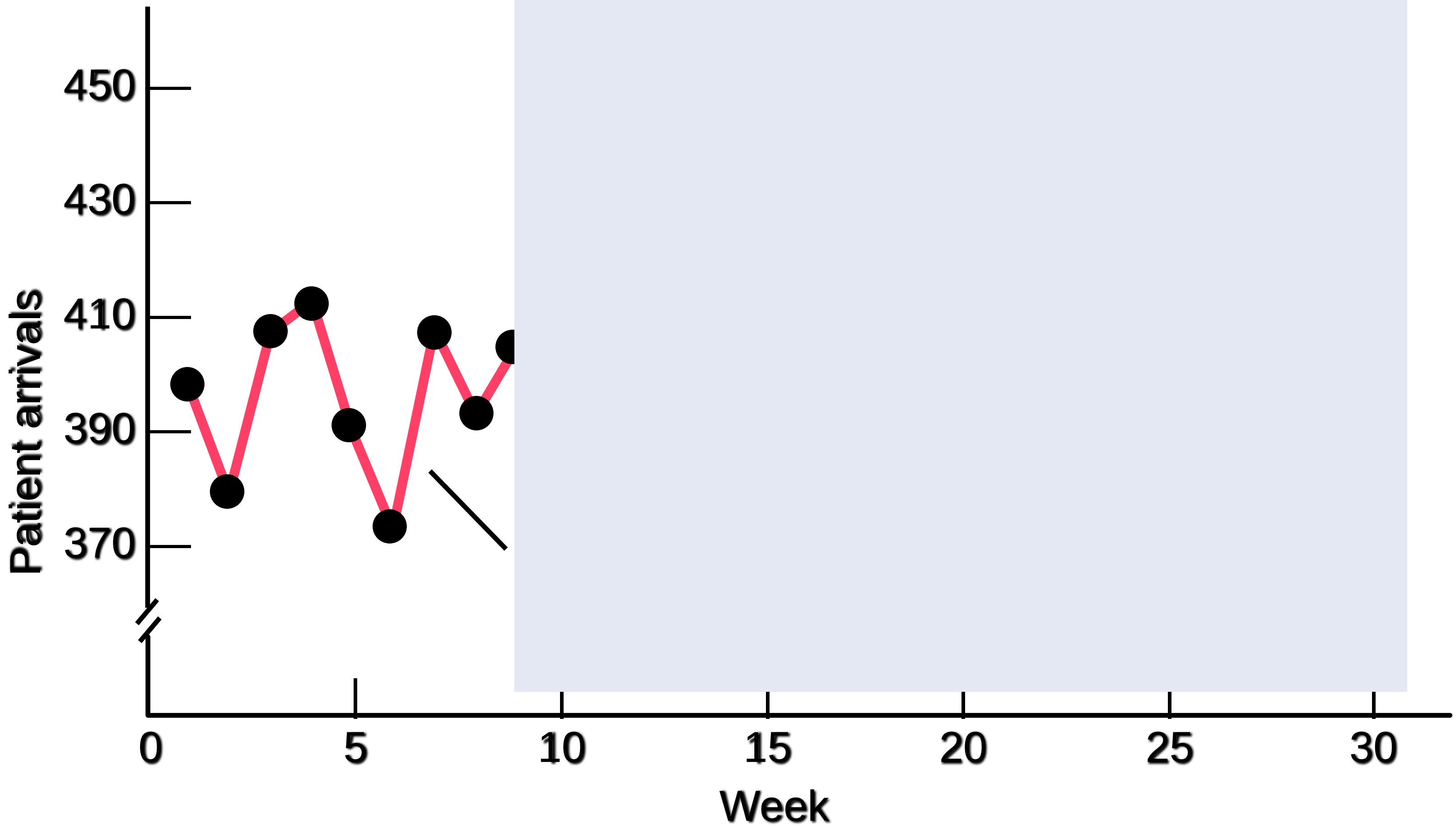
Time-Series Methods
Simple Moving Averages



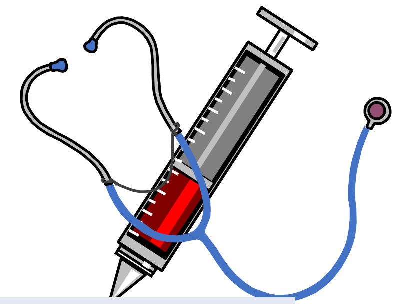


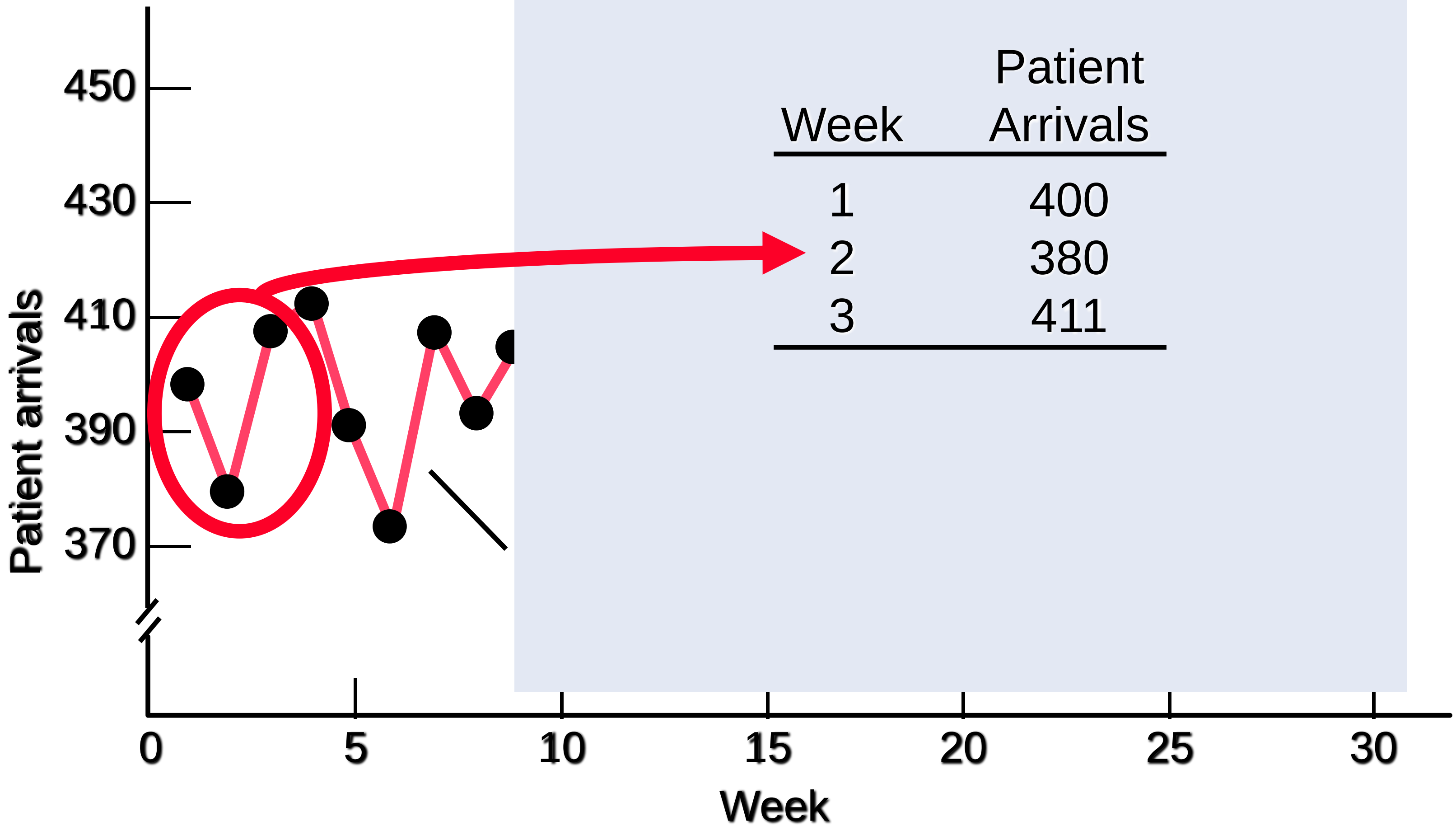
Time-Series Methods
Simple Moving Averages





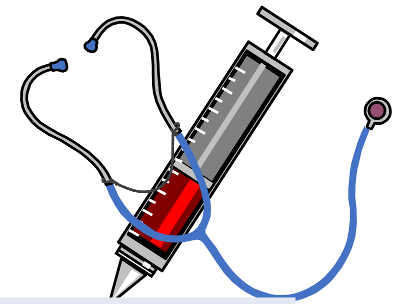
Time-Series Methods
Simple Moving Averages

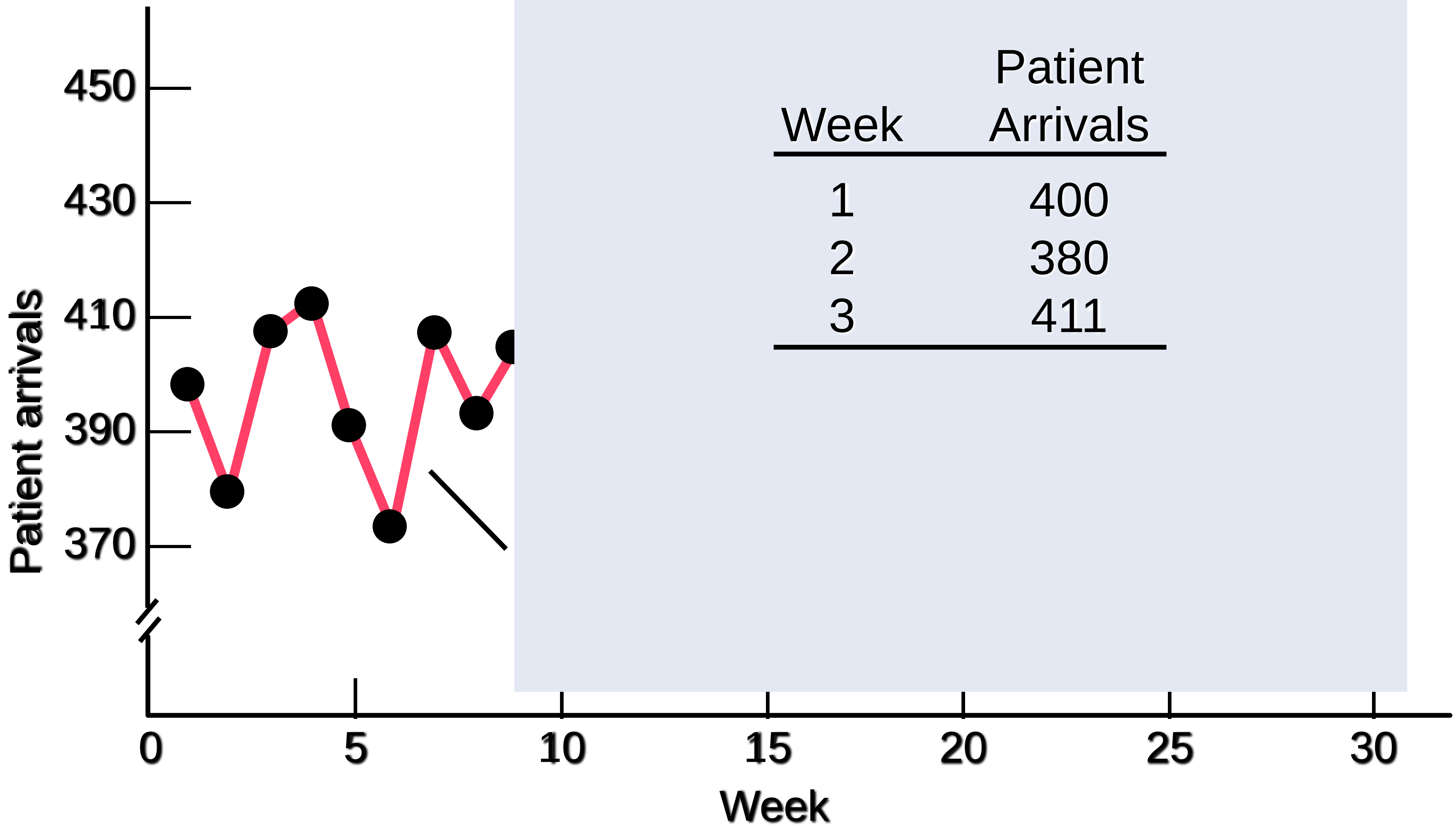




Week	Patient Arrivals
1	400
2	380
3	411

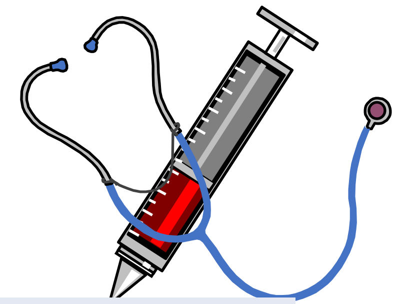
Time-Series Methods
Simple Moving Averages

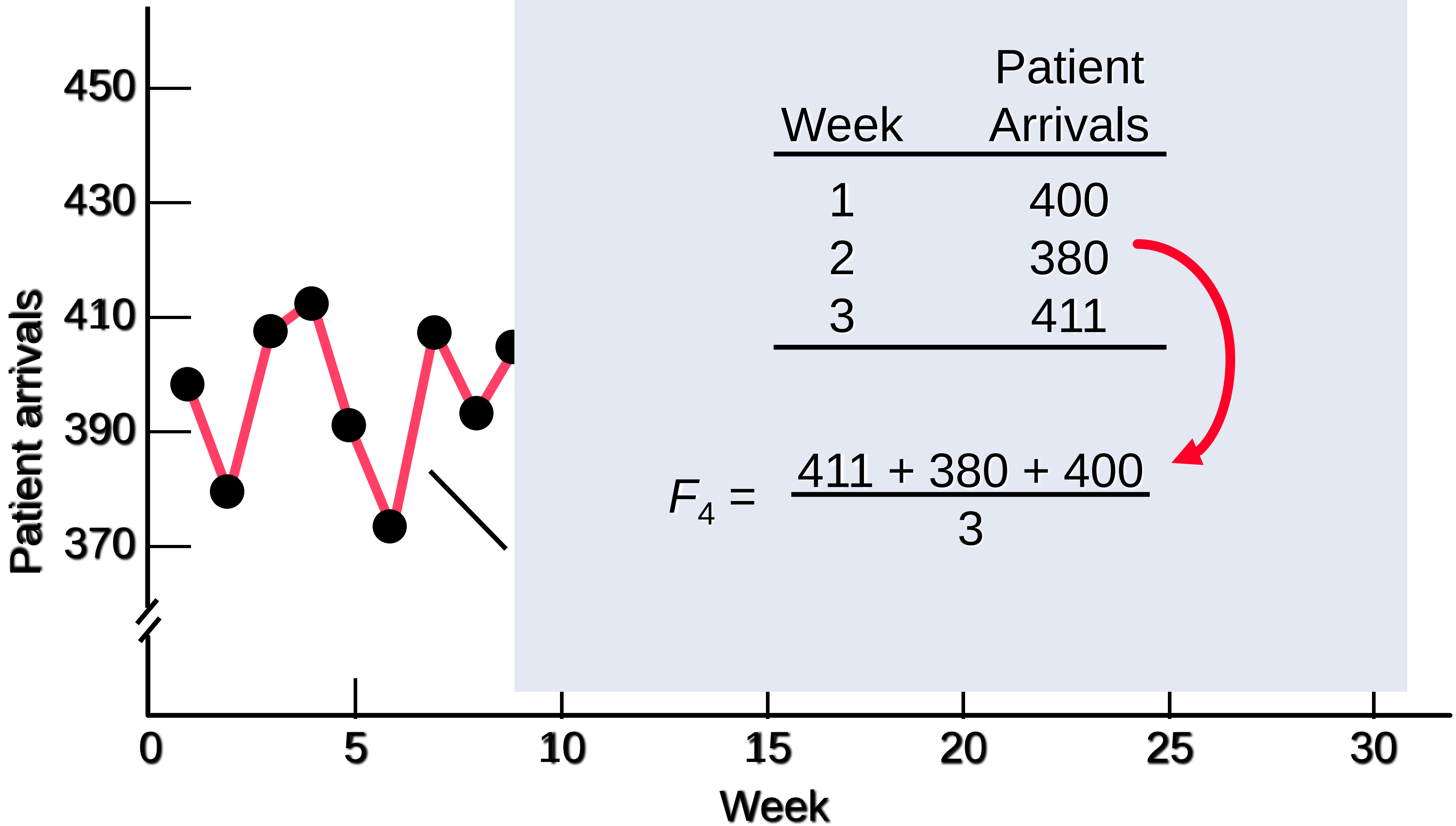




Week	Patient Arrivals
1	400
2	380
3	411

Time-Series Methods
Simple Moving Averages

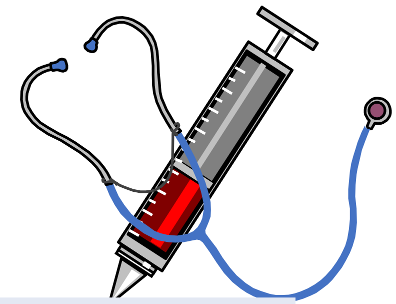


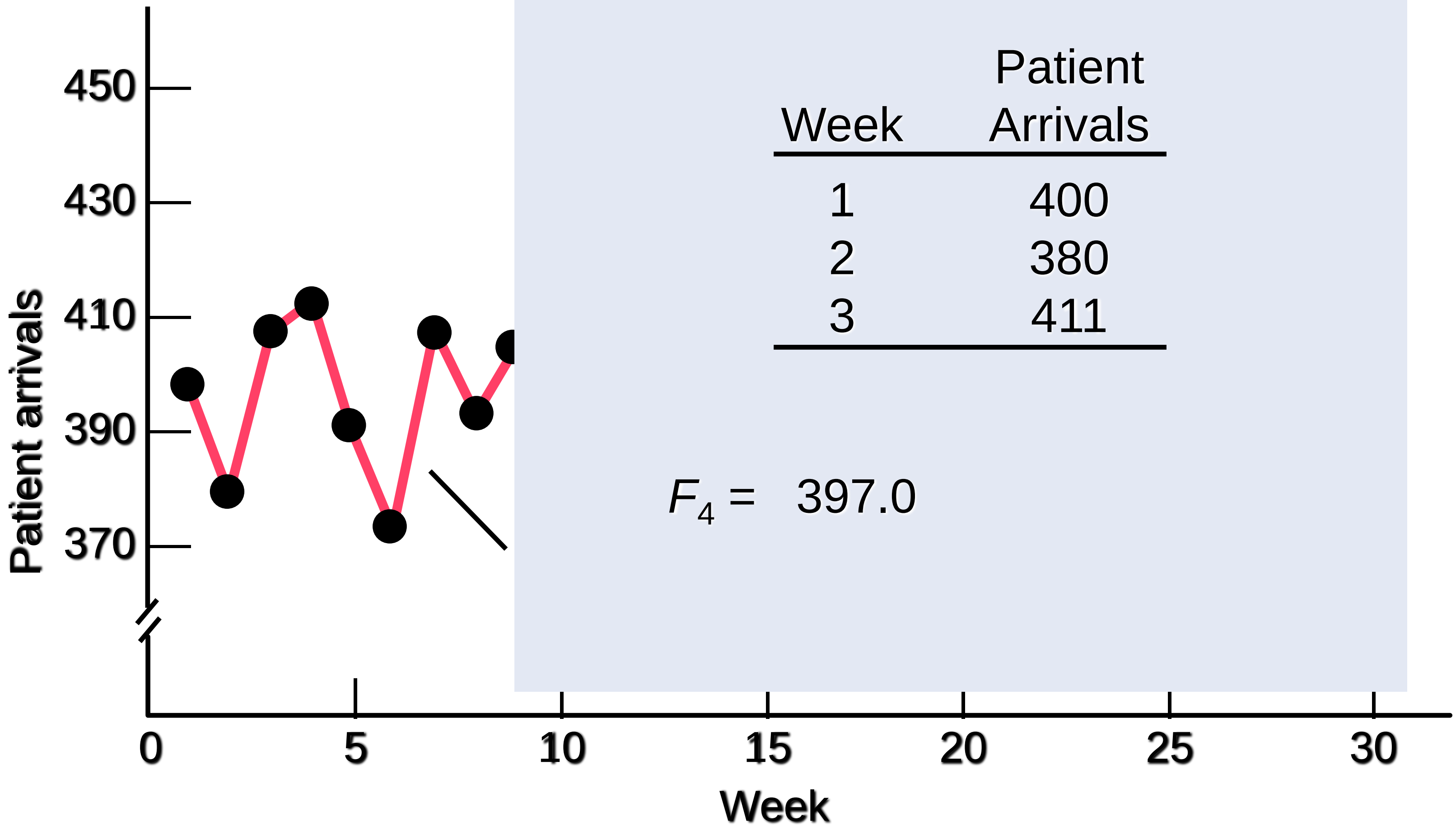


Week	Patient Arrivals
1	400
2	380
3	411

$$F_4 = \frac{411 + 380 + 400}{3}$$

Time-Series Methods
Simple Moving Averages

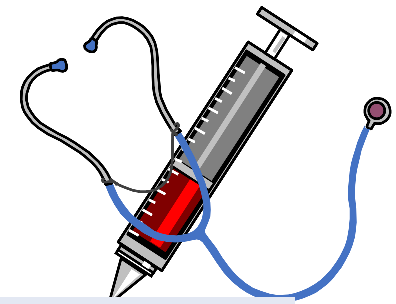


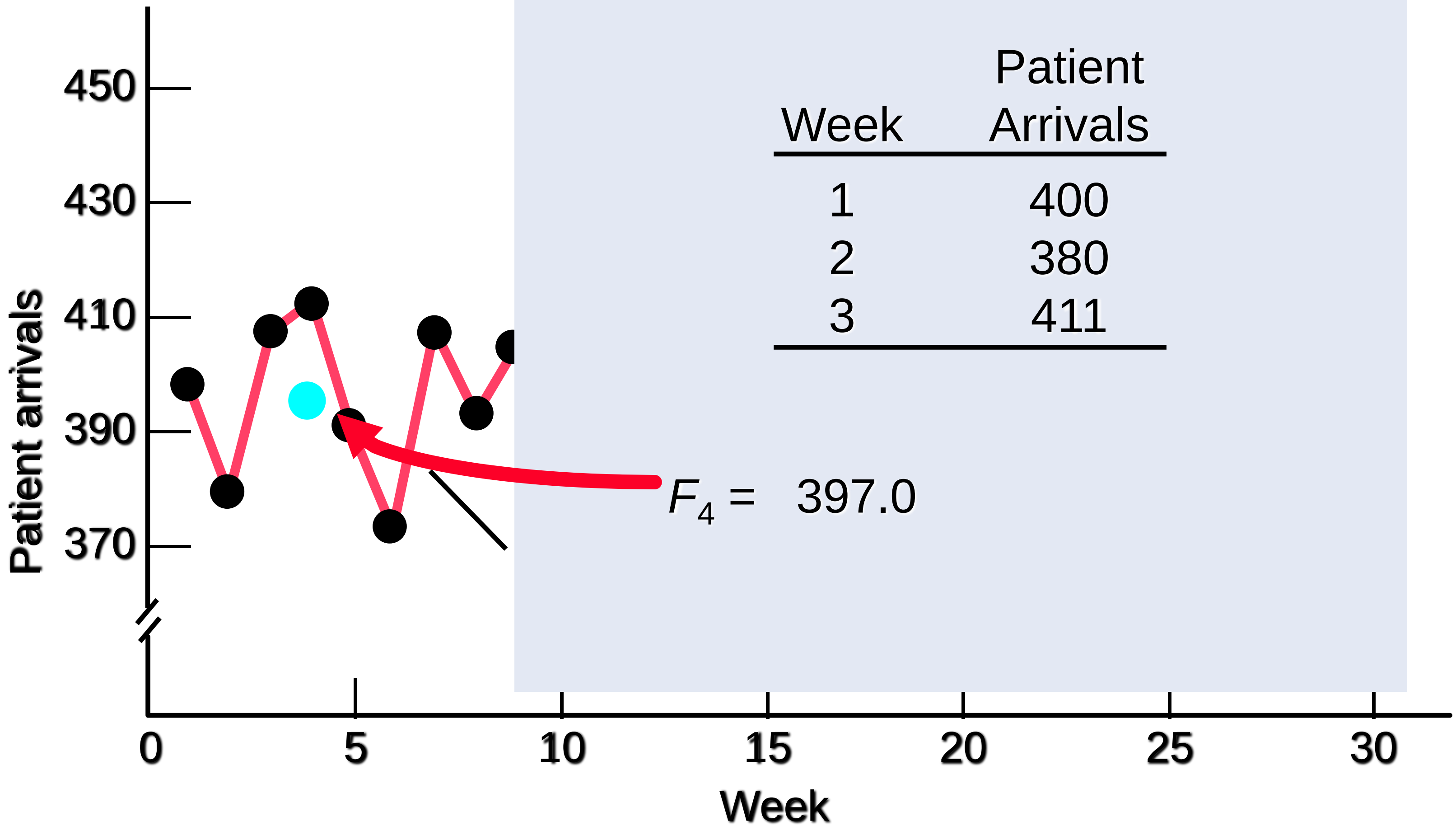


Week	Patient Arrivals
1	400
2	380
3	411

$F_4 = 397.0$

Time-Series Methods
Simple Moving Averages

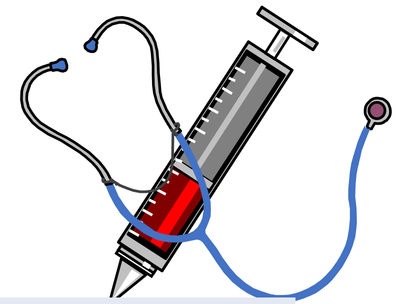


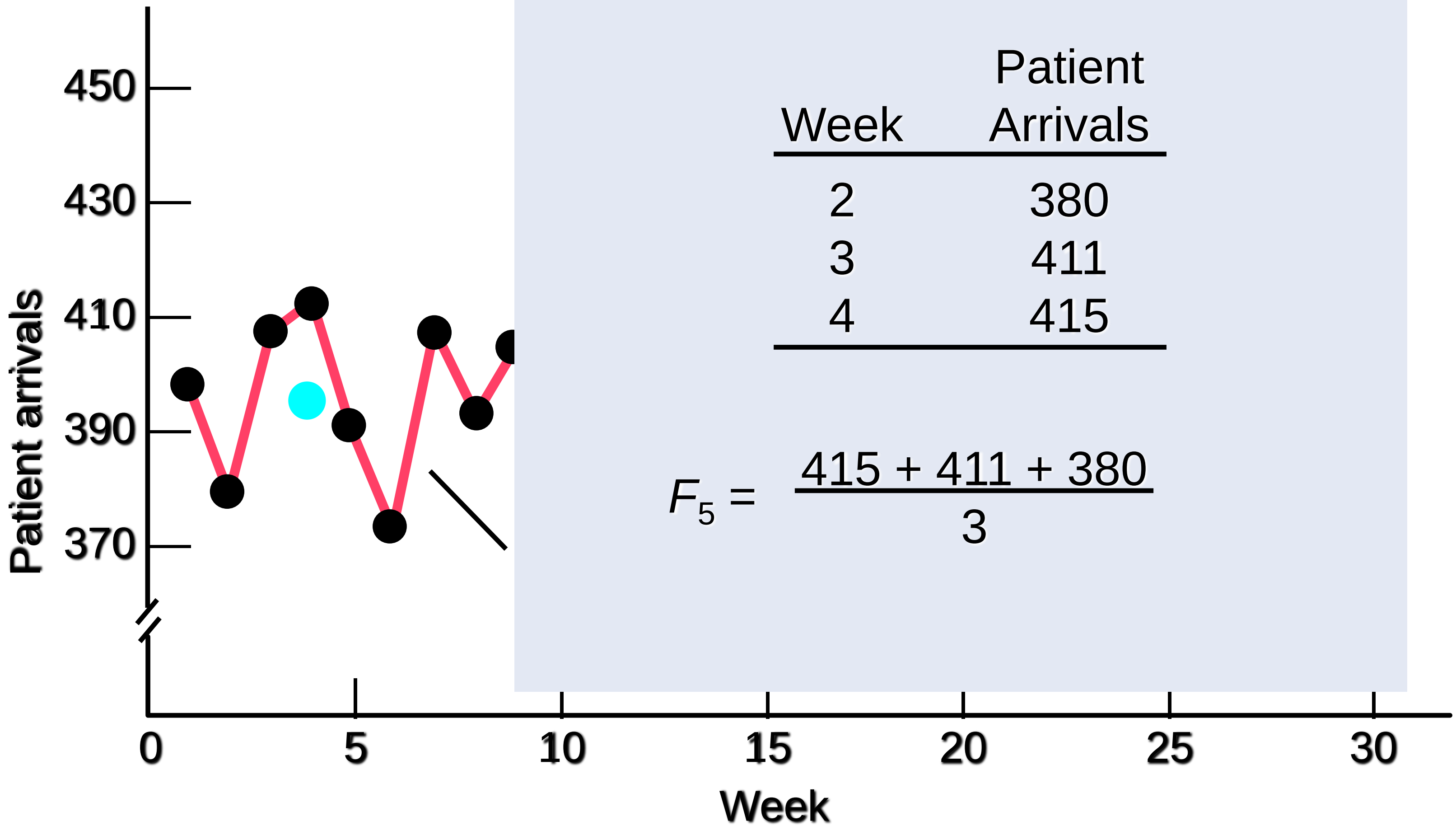


Week	Patient Arrivals
1	400
2	380
3	411

$F_4 =$ 397.0

Time-Series Methods
Simple Moving Averages

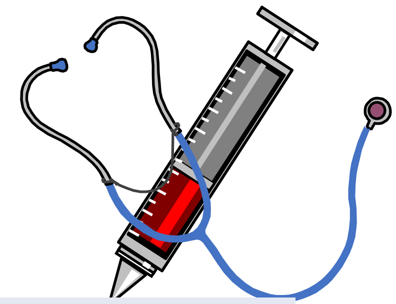


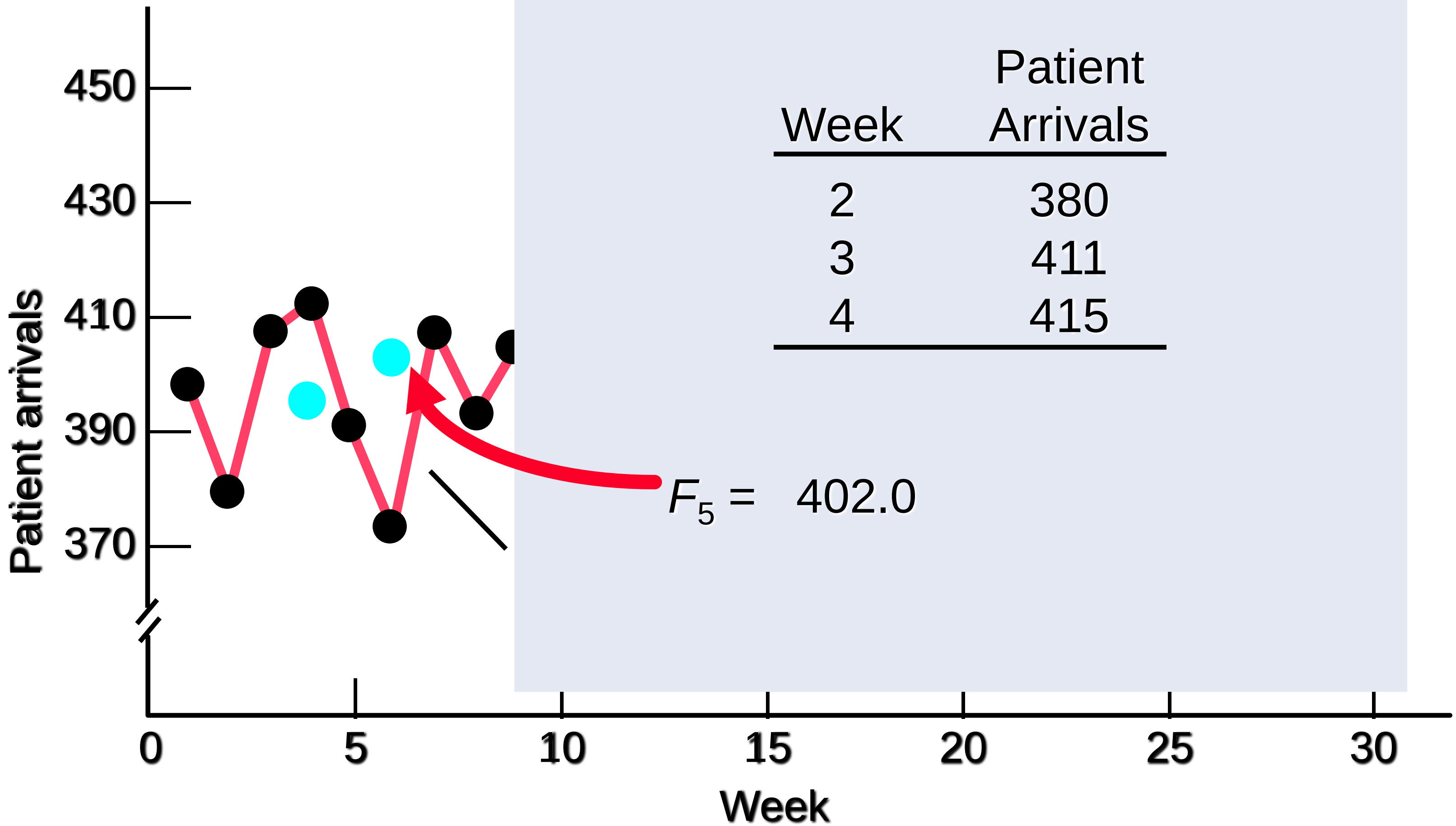


Week	Patient Arrivals
2	380
3	411
4	415

$$F_5 = \frac{415 + 411 + 380}{3}$$

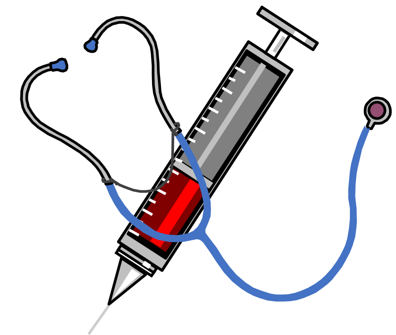
Time-Series Methods
Simple Moving Averages

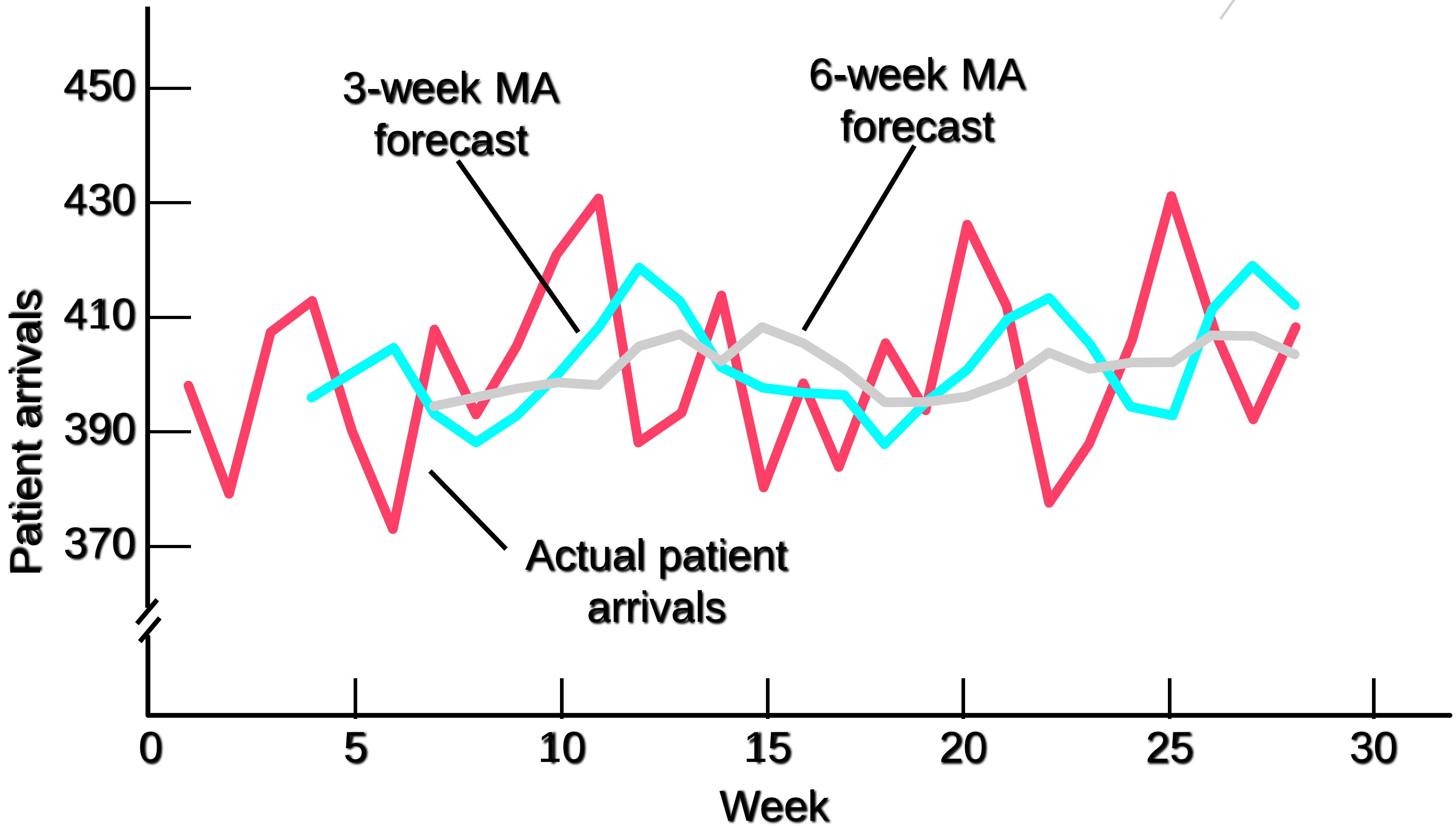




Week	Patient Arrivals
2	380
3	411
4	415

Time-Series Methods
Simple Moving Averages





DETERMINING THE WEIGHTED MOVING AVERAGE

Donna's Garden Supply (see Example 1) wants to forecast storage shed sales by weighting the past 3 months, with more weight given to recent data to make them more significant.

APPROACH ▶ Assign more weight to recent data, as follows:

WEIGHTS APPLIED	PERIOD
3	Last month
2	Two months ago
1	Three months ago
6	Sum of weights

Forecast for this month =

$$\frac{3 \times \text{Sales last mo.} + 2 \times \text{Sales 2 mos. ago} + 1 \times \text{Sales 3 mos. ago}}{\text{Sum of the weights}}$$

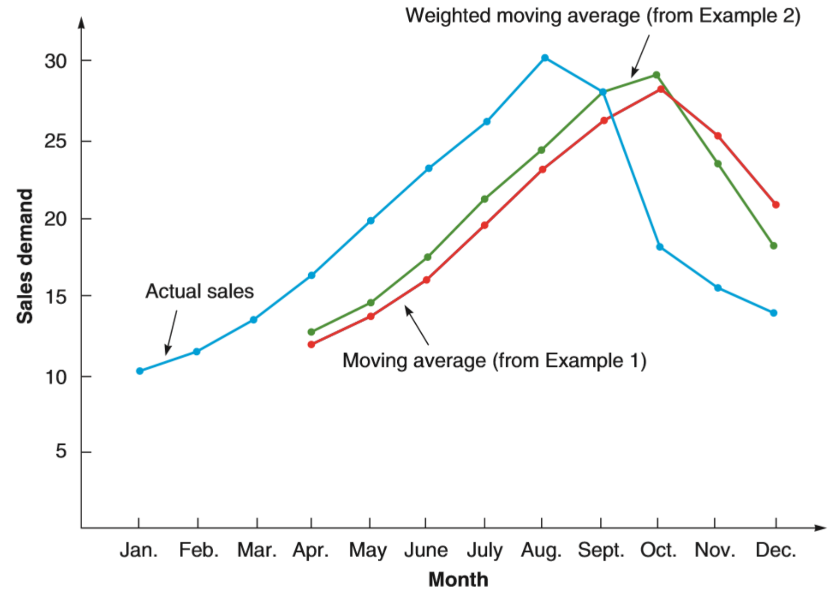
SOLUTION ▶ The results of this weighted-average forecast are as follows:

MONTH	ACTUAL SHED SALES	3-MONTH WEIGHTED MOVING AVERAGE
January	10	
February	12	
March	13	
April	16	$[(3 \times 13) + (2 \times 12) + (10)]/6 = 12\frac{1}{6}$
May	19	$[(3 \times 16) + (2 \times 13) + (12)]/6 = 14\frac{1}{3}$
June	23	$[(3 \times 19) + (2 \times 16) + (13)]/6 = 17$
July	26	$[(3 \times 23) + (2 \times 19) + (16)]/6 = 20\frac{1}{2}$
August	30	$[(3 \times 26) + (2 \times 23) + (19)]/6 = 23\frac{5}{6}$
September	28	$[(3 \times 30) + (2 \times 26) + (23)]/6 = 27\frac{1}{2}$
October	18	$[(3 \times 28) + (2 \times 30) + (26)]/6 = 28\frac{1}{3}$
November	16	$[(3 \times 18) + (2 \times 28) + (30)]/6 = 23\frac{1}{3}$
December	14	$[(3 \times 16) + (2 \times 18) + (28)]/6 = 18\frac{2}{3}$

WEIGHTED MOVING AVERAGE

Comparison

- Moving-average methods always lag behind when there is a trend present, as shown by the blue line (actual sales) for January through August.



Exponential Smoothing

Exponential smoothing is another weighted-moving-average forecasting method. It involves ver *little* record keeping of past data and is fairly easy to use. The basic exponential smoothin formula can be shown as follows:

$$\text{New forecast} = \text{Last period's forecast} + \alpha\,(\text{Last period's actual demand} - \text{Last period's forecast}) \qquad (4\text{-}3$$

where α is a weight, or **smoothing constant**, chosen by the forecaster, that has a value greate than or equal to 0 and less than or equal to 1. Equation (4-3) can also be written mathemati cally as:

$$F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1}) \qquad (4\text{-}4$$

where F_t = new forecast
F_{t-1} = previous period's forecast
α = smoothing (or weighting) constant ($0 \leq \alpha \leq 1$)
A_{t-1} = previous period's actual demand

Exponential Smoothing

Weighted exponential smoothing

It can be changed to give more weight to recent data (when a is high) or more weight to past data (when a is low).

	WEIGHT ASSIGNED TO				
SMOOTHING CONSTANT	MOST RECENT PERIOD (α)	2ND MOST RECENT PERIOD $\alpha(1-\alpha)$	3RD MOST RECENT PERIOD $\alpha(1-\alpha)^2$	4TH MOST RECENT PERIOD $\alpha(1-\alpha)^2$	5TH MOST RECENT PERIOD $\alpha(1-\alpha)^4$
$\alpha = .1$	.1	.09	.081	.073	.066
$\alpha = .5$	.5	.25	.125	.063	.031

- Better trend identification
- Demand and forecasts are smoothed

To improve our forecast, let us illustrate a more complex exponential smoothing model, one that adjusts for trend. The idea is to compute an exponentially smoothed average of the data and then adjust for positive or negative lag in trend. The new formula is:

$$\text{Forecast including trend } (FIT_t) = \text{Exponentially smoothed forecast average } (F_t) + \text{Exponentially smoothed trend } (T_t) \qquad (4\text{-}8)$$

With trend-adjusted exponential smoothing, estimates for both the average and the trend are smoothed. This procedure requires two smoothing constants: α for the average and β for the trend. We then compute the average and trend each period:

$$F_t = \alpha(\text{Actual demand last period}) + (1 - \alpha)(\text{Forecast last period} + \text{Trend estimate last period})$$

or:

$$F_t = \alpha(A_{t-1}) + (1 - \alpha)(F_{t-1} + T_{t-1}) \qquad (4\text{-}9)$$

$$T_t = \beta(\text{Forecast this period} - \text{Forecast last period}) + (1 - \beta)(\text{Trend estimate last period})$$

or:

$$T_t = \beta(F_t - F_{t-1}) + (1 - \beta)T_{t-1} \qquad (4\text{-}10)$$

where F_t = exponentially smoothed forecast average of the data series in period t
T_t = exponentially smoothed trend in period t
A_t = actual demand in period t
α = smoothing constant for the average ($0 \leq \alpha \leq 1$)
β = smoothing constant for the trend ($0 \leq \beta \leq 1$)

Changing trend

MONTH	ACTUAL DEMAND	FORECAST (F_t) FOR MONTHS 1–5
1	100	$F_1 = 100$ (given)
2	200	$F_2 = F_1 + \alpha(A_1 - F_1) = 100 + .4(100 - 100) = 100$
3	300	$F_3 = F_2 + \alpha(A_2 - F_2) = 100 + .4(200 - 100) = 140$
4	400	$F_4 = F_3 + \alpha(A_3 - F_3) = 140 + .4(300 - 140) = 204$
5	500	$F_5 = F_4 + \alpha(A_4 - F_4) = 204 + .4(400 - 204) = 282$

To improve our forecast, let us illustrate a more complex exponential smoothing model, one that adjusts for trend. The idea is to compute an exponentially smoothed average of the data and then adjust for positive or negative lag in trend. The new formula is:

$$\text{Forecast including trend } (FIT_t) = \text{Exponentially smoothed forecast average } (F_t) + \text{Exponentially smoothed trend } (T_t) \qquad \text{(4-8)}$$

With trend-adjusted exponential smoothing, estimates for both the average and the trend are smoothed. This procedure requires two smoothing constants: α for the average and β for the trend. We then compute the average and trend each period:

$$F_t = \alpha(\text{Actual demand last period}) + (1 - \alpha)(\text{Forecast last period} + \text{Trend estimate last period})$$

or:

$$F_t = \alpha(A_{t-1}) + (1 - \alpha)(F_{t-1} + T_{t-1}) \qquad \text{(4-9)}$$

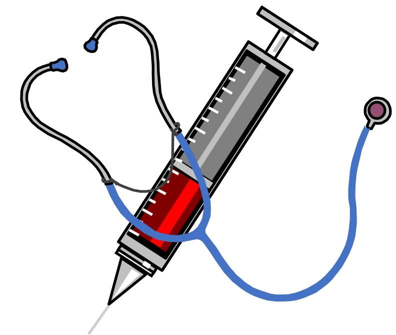
$$T_t = \beta(\text{Forecast this period} - \text{Forecast last period}) + (1 - \beta)(\text{Trend estimate last period})$$

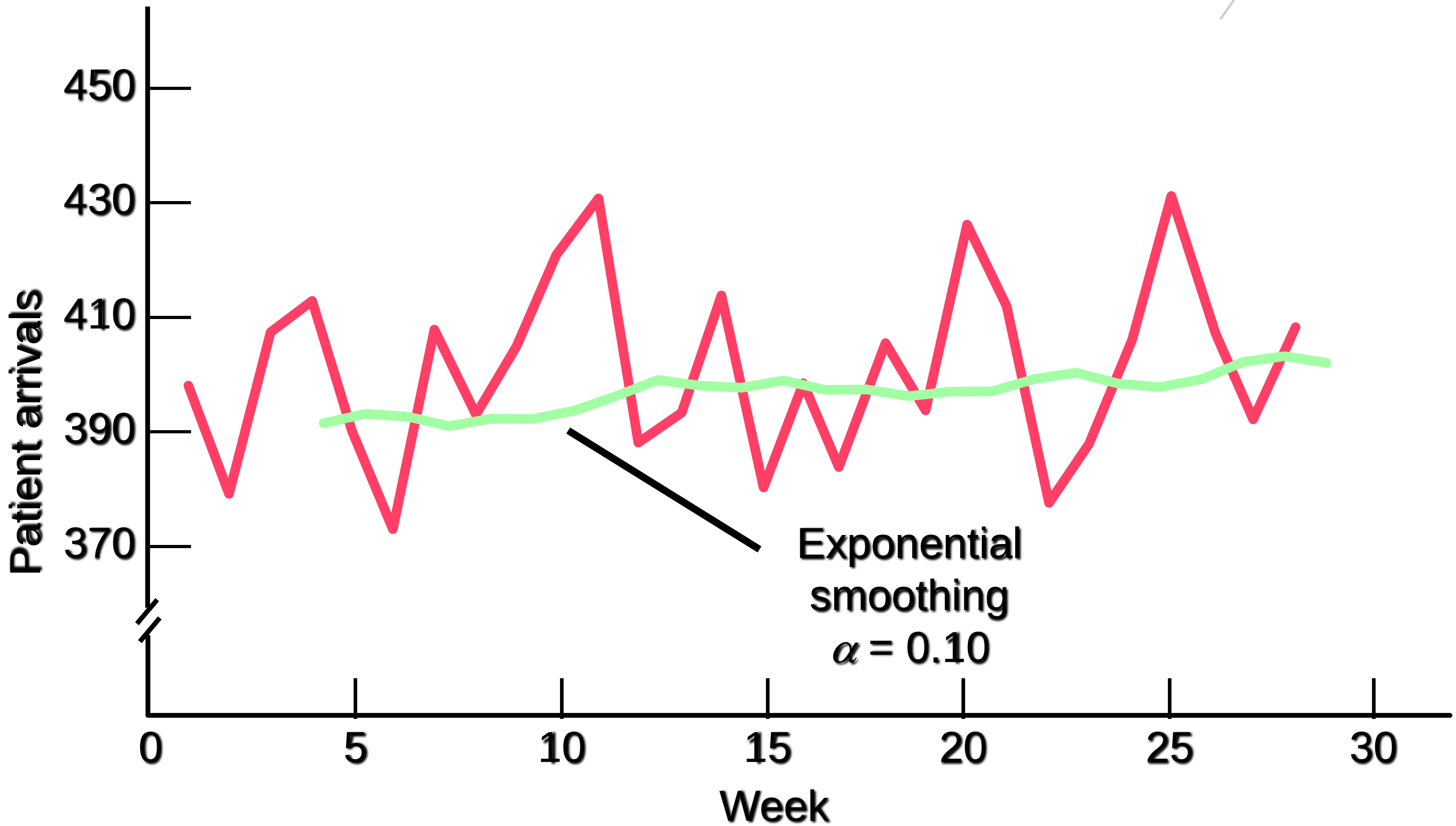
or:

$$T_t = \beta(F_t - F_{t-1}) + (1 - \beta)T_{t-1} \qquad \text{(4-10)}$$

where F_t = exponentially smoothed forecast average of the data series in period t
T_t = exponentially smoothed trend in period t
A_t = actual demand in period t
α = smoothing constant for the average $(0 \le \alpha \le 1)$
β = smoothing constant for the trend $(0 \le \beta \le 1)$

Time-Series Methods
Exponential Smoothing





Exponential Smoothing with Trend Adjustment Example

MONTH (t)	ACTUAL DEMAND (A_t)	MONTH (t)	ACTUAL DEMAND (A_t)
1	12	6	21
2	17	7	31
3	20	8	28
4	19	9	36
5	24	10	?

$\alpha = .2$ $\beta = .4$

Exponential Smoothing with Trend Adjustment Example (1 of 5)

Table 4.2 Forecast with α = .2 and β = .4

MONTH	ACTUAL DEMAND	SMOOTHED FORECAST AVERAGE, F_t	SMOOTHED TREND, T_t	FORECAST INCLUDING TREND, FIT_t
1	12	11	2	13.00
2	17	12.80		
3	20			
4	19			
5	24			
6	21			
7	31			
8	28			
9	36			
10	—			

Step 1: Average for Month 2

$$F_2 = \alpha A_1 + (1 - \alpha)(F_1 + T_1)$$

$$F_2 = (.2)(12) + (1 - .2)(11 + 2)$$

$$= 2.4 + (.8)(13) = 2.4 + 10.4$$

$$= 12.8 \text{ units}$$

Exponential Smoothing with Trend Adjustment Example (2 of 5)

Table 4.2 Forecast with α = .2 and β = .4

MONTH	ACTUAL DEMAND	SMOOTHED FORECAST AVERAGE, F_t	SMOOTHED TREND, T_t	FORECAST INCLUDING TREND, FIT_t
1	12	11	2	13.00
2	17	12.80	1.92	
3	20			
4	19			
5	24			
6	21			
7	31			
8	28			
9	36			
10	—			

Step 2: Trend for Month 2

$$T_2 = \beta(F_2 - F_1) + (1 - \beta)T_1$$

$$T_2 = (.4)(12.8 - 11) + (1 - .4)(2)$$

$$= .72 + 1.2 = 1.92 \text{ units}$$

Exponential Smoothing with Trend Adjustment Example (3 of 5)

Table 4.2 Forecast with α = .2 and β = .4

MONTH	ACTUAL DEMAND	SMOOTHED FORECAST AVERAGE, F_t	SMOOTHED TREND, T_t	FORECAST INCLUDING TREND, FIT_t
1	12	11	2	13.00
2	17	12.80	1.92	14.72
3	20			
4	19			
5	24			
6	21			
7	31			
8	28			
9	36			
10	—			

Step 3: Calculate *FIT* for Month 2

$$FIT_2 = F_2 + T_2$$

$$FIT_2 = 12.8 + 1.92$$

$$= 14.72 \text{ units}$$

Exponential Smoothing with Trend Adjustment Example (4 of 5)

Table 4.2 Forecast with α = .2 and β = .4

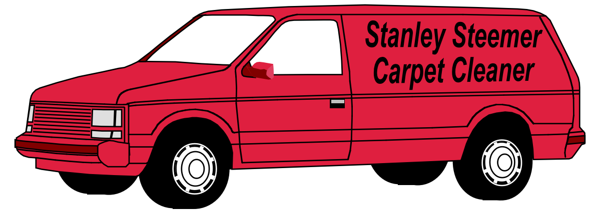
MONTH	ACTUAL DEMAND	SMOOTHED FORECAST AVERAGE, F_t	SMOOTHED TREND, T_t	FORECAST INCLUDING TREND, FIT_t
1	12	11	2	13.00
2	17	12.80	1.92	14.72
3	20	15.18	2.10	17.28
4	19	17.82	2.32	20.14
5	24	19.91	2.23	22.14
6	21	22.51	2.38	24.89
7	31	24.11	2.07	26.18
8	28	27.14	2.45	29.59
9	36	29.28	2.32	31.60
10		32.48	2.68	35.16

Seasonal indices

1. Find the *average historical demand each season* (or month in this case) by summing the demand for that month in each year and dividing by the number of years of data available. For example, if, in January, we have seen sales of 8, 6, and 10 over the past 3 years, average January demand equals $(8 + 6 + 10)/3 = 8$ units.
2. Compute the *average demand over all months* by dividing the total average annual demand by the number of seasons. For example, if the total average demand for a year is 120 units and there are 12 seasons (each month), the average monthly demand is $120/12 = 10$ units.
3. Compute a *seasonal index* for each season by dividing that *month's* historical average demand (from Step 1) by the average demand over all months (from Step 2). For example, if the average historical January demand over the past 3 years is 8 units and the average demand over all months is 10 units, the seasonal index for January is $8/10 = .80$. Likewise, a seasonal index of 1.20 for February would mean that February's demand is 20% larger than the average demand over all months.
4. Estimate next year's total annual demand.
5. Divide this estimate of total annual demand by the number of seasons, then multiply it by the seasonal index for each month. This provides the *seasonal forecast*.

Time-Series Methods

Seasonal Influences



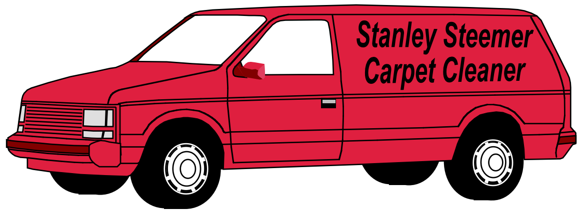
Quarter	Year 1	Year 2	Year 3	Year 4
1	45	70	100	100
2	335	370	585	725
3	520	590	830	1160
4	100	170	285	215
Total	1000	1200	1800	2200

Seasonal variations

Regular upward or downward movements in a time series that tie to recurring events.

Time-Series Methods

Seasonal Influences



Period: Quarters

Starting Year: 1

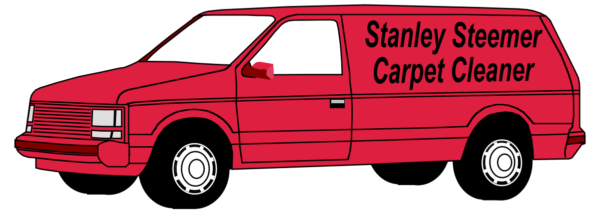
Years: 4

Computed Forecast Demand for Year 5: 2600

User-supplied Forecast Demand for Year 5: 2600

	Year			
Quarter	1	2	3	4
1	45	70	100	100
2	335	370	585	725
3	520	590	830	1160
4	100	170	285	215

Time-Series Methods
Seasonal Influences



Period — Qua

Starting Year

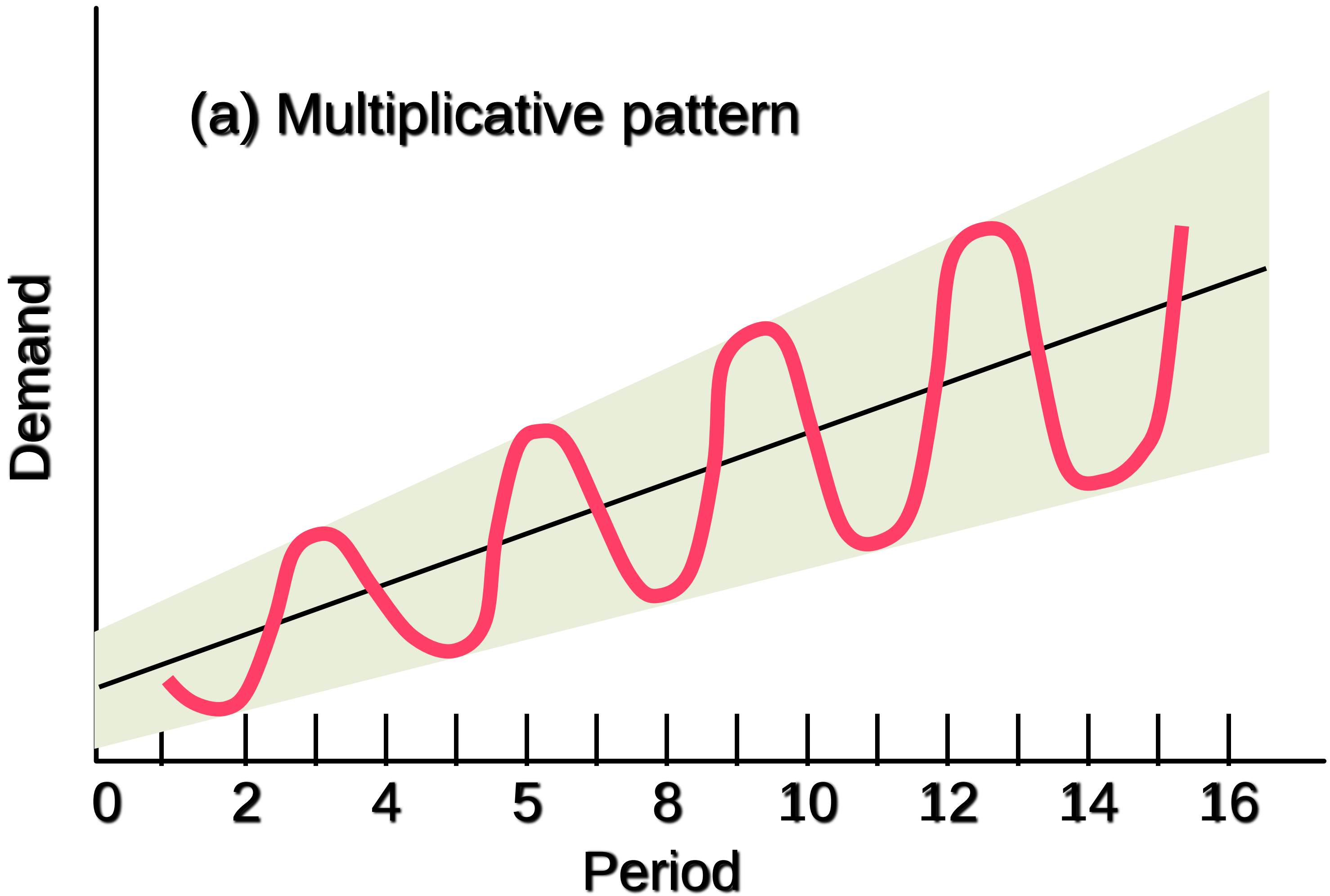
Computed Forecast

User-supplied Foreca

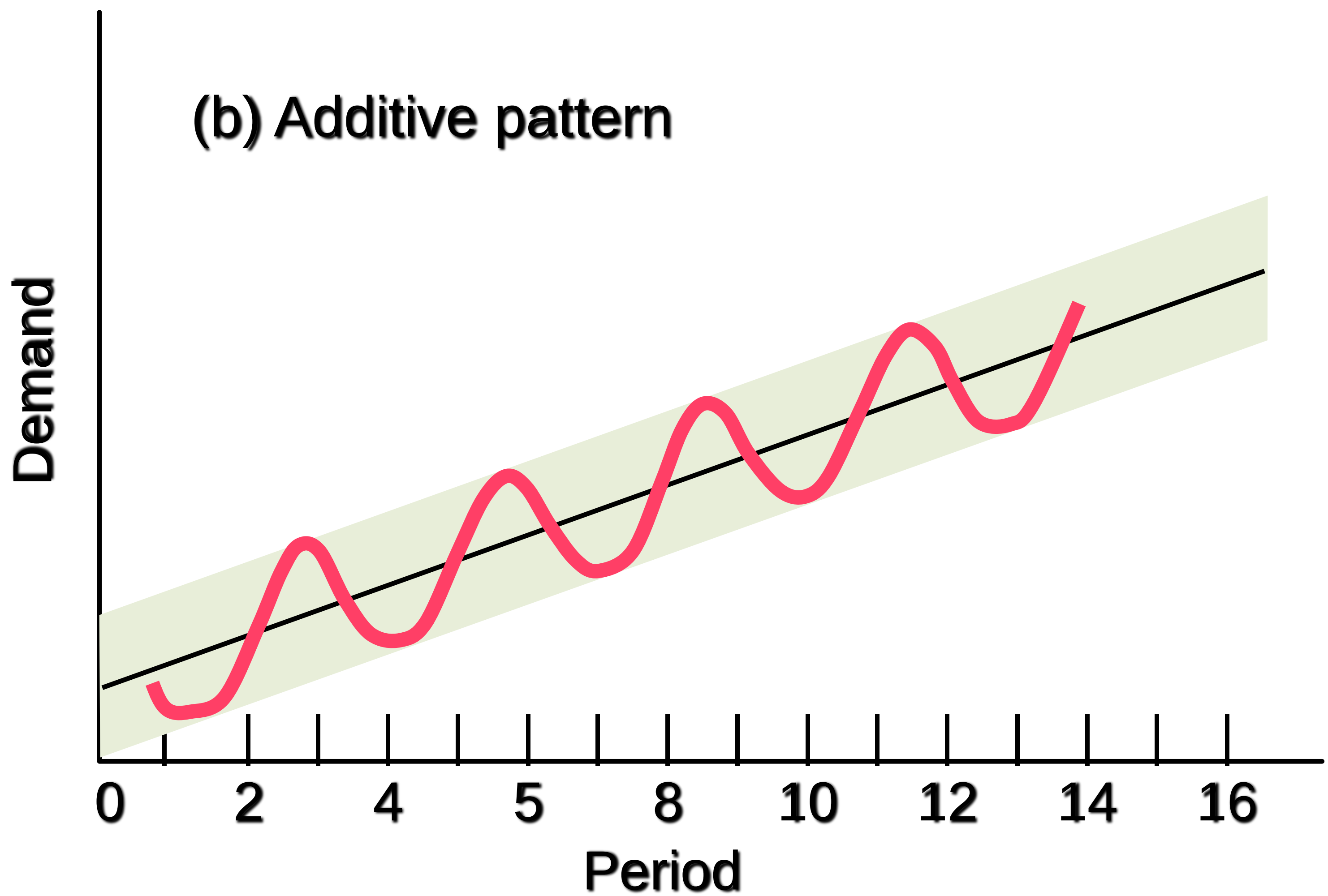
Quarter	Seasonal Index	Forecast
1	0.2043	132.795
2	1.2979	843.635
3	2.0001	1300.065
4	0.4977	323.505

	Year			
Quarter	1	2	3	4
1	45	70	100	100
2	335	370	585	725
3	520	590	830	1160
4	100	170	285	215

Seasonal Patterns



Seasonal Patterns



Seasonal Index Example (1 of 6)

DEMAND						
MONTH	YEAR 1	YEAR 2	YEAR 3	AVERAGE PERIOD DEMAND	AVERAGE MONTHLY DEMAND	SEASONAL INDEX
Jan	80	85	105	90		
Feb	70	85	85	80		
Mar	80	93	82	85		
Apr	90	95	115	100		
May	113	125	131	123		
June	110	115	120	115		
July	100	102	113	105		
Aug	88	102	110	100		
Sept	85	90	95	90		
Oct	77	78	85	80		
Nov	75	82	83	80		
Dec	82	78	80	80		
Total average annual demand =				1,128		

Seasonal Index Example (2 of 6)

	DEMAND					
MONTH	YEAR 1	YEAR 2	YEAR 3	AVERAGE PERIOD DEMAND	AVERAGE MONTHLY DEMAND	SEASONAL INDEX
Jan	80	85	105	90	94	
Feb	70	85	85	80	94	
Mar	[illegible]	[illegible]	[illegible]	[illegible]	94	
Apr	[illegible]	[illegible]	[illegible]	[illegible]	94	
May	[illegible]	[illegible]	[illegible]	[illegible]	94	
June	[illegible]	[illegible]	[illegible]	[illegible]	94	
July	100	102	113	105	94	
Aug	88	102	110	100	94	
Sept	85	90	95	90	94	
Oct	77	78	85	80	94	
Nov	75	82	83	80	94	
Dec	82	78	80	80	94	
			Total average annual demand =	1,128		

$$\text{Average monthly demand} = \frac{1{,}128}{12 \text{ months}} = 94$$

Seasonal Index Example (3 of 6)

DEMAND						
MONTH	YEAR 1	YEAR 2	YEAR 3	AVERAGE PERIOD DEMAND	AVERAGE MONTHLY DEMAND	SEASONAL INDEX
Jan	80	85	105	90	94	.957 (= 90/94)
Feb	70	85	85	80	94	
Mar	80	93	82	85	94	
Apr	90	95	115	100	94	
Sept	85	90	95	90	94	
Oct	77	78	85	80	94	
Nov	75	82	83	80	94	
Dec	82	78	80	80	94	
Total average annual demand =				1,128		

$$\text{Seasonal index} = \frac{\text{Average monthly demand for past 3 years}}{\text{Average monthly demand}}$$

Seasonal Index Example (4 of 6)

DEMAND						
MONTH	YEAR 1	YEAR 2	YEAR 3	AVERAGE PERIOD DEMAND	AVERAGE MONTHLY DEMAND	SEASONAL INDEX
Jan	80	85	105	90	94	.957 (= 90/94)
Feb	70	85	85	80	94	.851 (= 80/94)
Mar	80	93	82	85	94	.904 (= 85/94)
Apr	90	95	115	100	94	1.064 (= 100/94)
May	113	125	131	123	94	1.309 (= 123/94)
June	110	115	120	115	94	1.223 (= 115/94)
July	100	102	113	105	94	1.117 (= 105/94)
Aug	88	102	110	100	94	1.064 (= 100/94)
Sept	85	90	95	90	94	.957 (= 90/94)
Oct	77	78	85	80	94	.851 (= 80/94)
Nov	75	82	83	80	94	.851 (= 80/94)
Dec	82	78	80	80	94	.851 (= 80/94)
Total average annual demand =				1,128		

Seasonal Index Example (5 of 6)

Seasonal forecast for Year 4

MONTH	DEMAND	MONTH	DEMAND
Jan	$\frac{1,200}{12} \times .957 = 96$	July	$\frac{1,200}{12} \times 1.117 = 112$
Feb	$\frac{1,200}{12} \times .851 = 85$	Aug	$\frac{1,200}{12} \times 1.064 = 106$
Mar	$\frac{1,200}{12} \times .904 = 90$	Sept	$\frac{1,200}{12} \times .957 = 96$
Apr	$\frac{1,200}{12} \times 1.064 = 106$	Oct	$\frac{1,200}{12} \times .851 = 85$
May	$\frac{1,200}{12} \times 1.309 = 131$	Nov	$\frac{1,200}{12} \times .851 = 85$
June	$\frac{1,200}{12} \times 1.223 = 122$	Dec	$\frac{1,200}{12} \times .851 = 85$

USING REGRESSION ANALYSIS FOR FORECASTING

- We can use the same mathematical model that we employed in the least-squares method of trend projection to perform a linear-regression analysis.
- The dependent variables that we want to forecast will still be n y . But now the independent variable, x , need no longer be time.
- We use the equation: n y = a + bx where n y = value of the dependent variable (in our example, sales) a = y - axis intercept b = slope of the regression line x = independent variable

We now deal with the same mathematical model that we saw earlier, the least-squares method. But we use any potential “cause-and-effect”

Trend projections

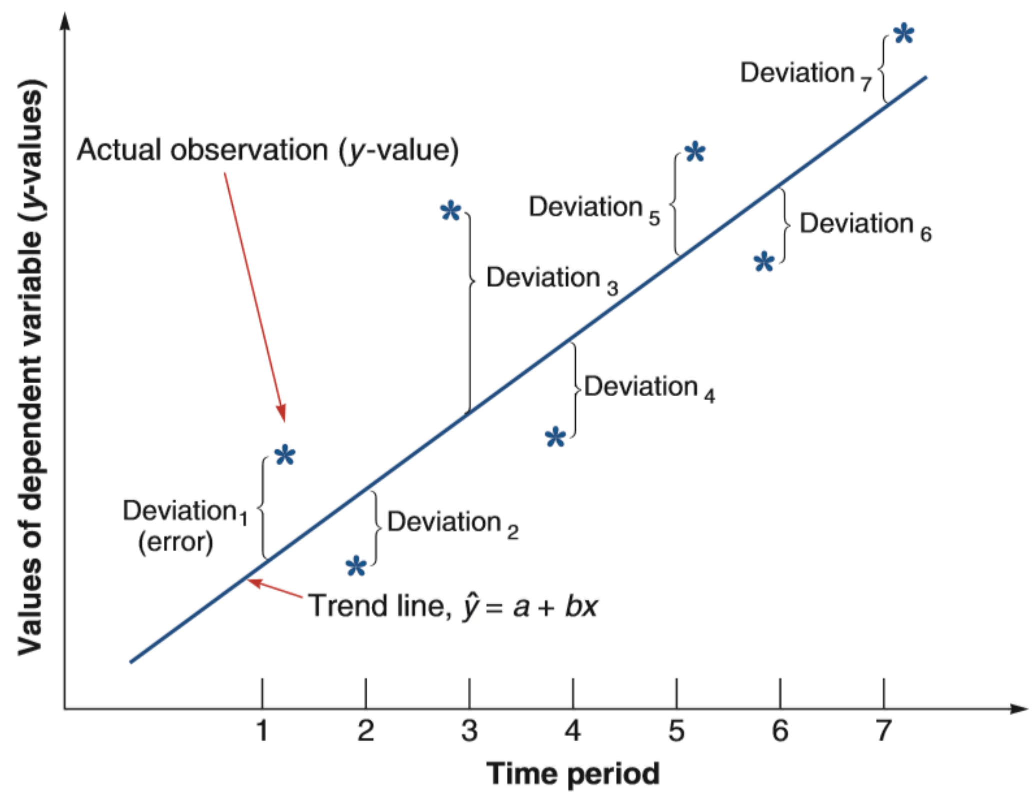
And a is = (average y) – b (average x)

$$\hat{y} = a + bx$$

$$b = \frac{\sum xy - nx\,y}{\sum x^2 - n\bar{x}^2}$$

Figure **4.4**

The Least-Squares Method for Finding the Best-Fitting Straight Line, Where the Asterisks Are the Locations of the Seven Actual Observations or Data Points



Least Squares Example

YEAR	ELECTRICAL POWER DEMAND	YEAR	ELECTRICAL POWER DEMAND
1	74	5	105
2	79	6	142
3	80	7	122
4	90		

Least Squares Example

YEAR (x)	ELECTRICAL POWER DEMAND (y)	x^2	xy
1	74	1	74
2	79	4	158
3	80	9	240
4	90	16	360
5	105	25	525
6	142	36	852
7	122	49	854
Σx = 28	Σy = 692	Σx^2 = 140	Σxy = 3,063

$$\bar{x} = \frac{\sum x}{n} = \frac{28}{7} = 4 \qquad \bar{y} = \frac{\sum y}{n} = \frac{692}{7} = 98.86$$

Least Squares Example

$\sum x = 28$	$\sum y = 692$	$\sum x^2 = 140$	$\sum xy = 3,063$

$$b = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2} = \frac{3,063 - (7)(4)(98.86)}{140 - (7)(4^2)} = \frac{295}{28} = 10.54$$

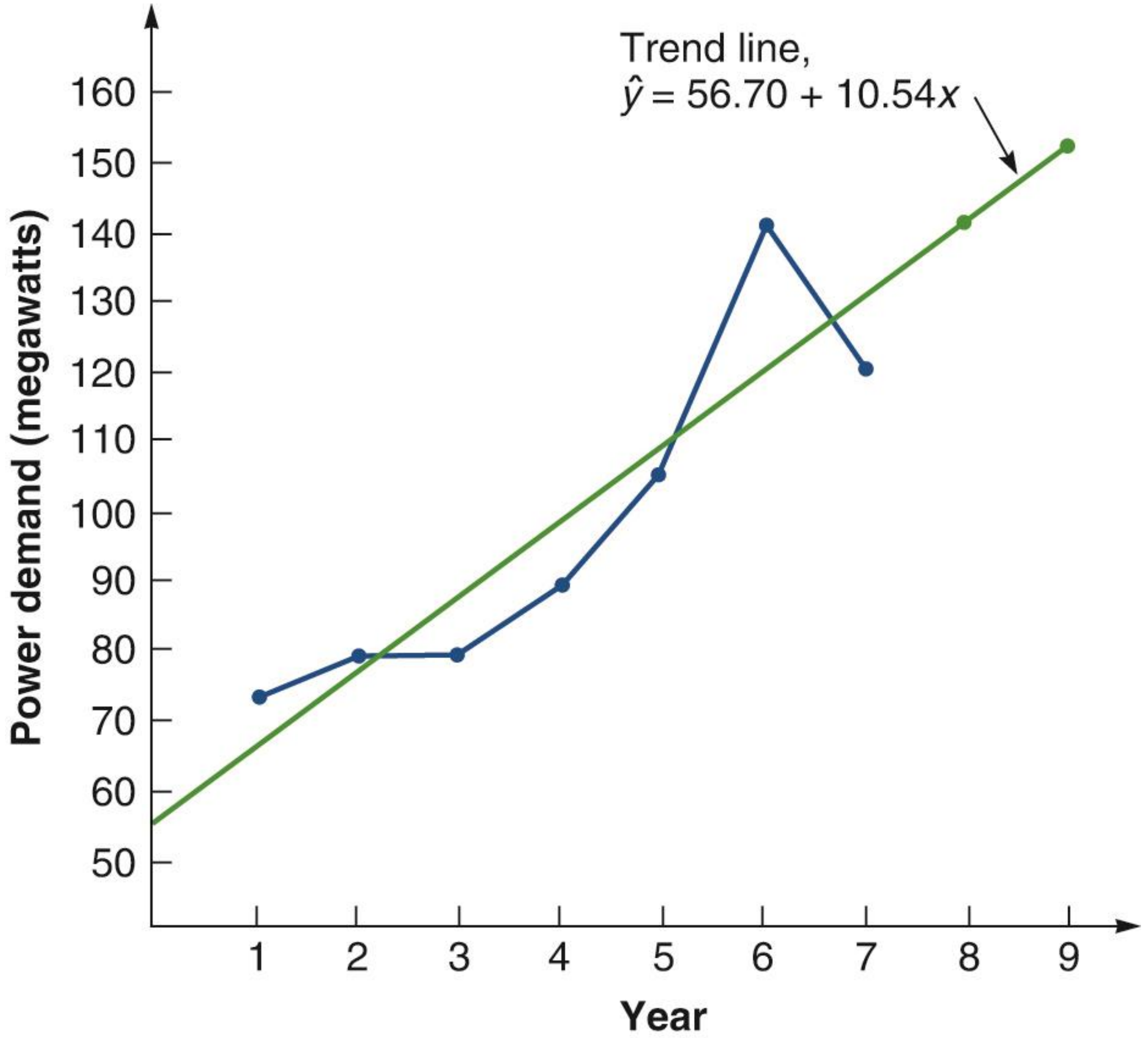
$$a = \bar{y} - b\bar{x} = 98.86 - 10.54(4) = 56.70$$

Thus, $\hat{y} = 56.70 + 10.54x$

Demand in year 8 = 56.70 + 10.54(8)
= 141.02, or 141 megawatts

Figure 4.5

Least Squares Example



Least Squares Requirements

- We always plot the data to insure a linear relationship
- We do not predict time periods far beyond the database
- Deviations around the least squares line are assumed to be random

Forecast errors

$$\text{Tracking signal} = \frac{\text{Cumulative error}}{\text{MAD}}$$

$$= \frac{\sum(\text{Actual demand in period } i - \text{Forecast demand in period } i)}{\text{MAD}}$$

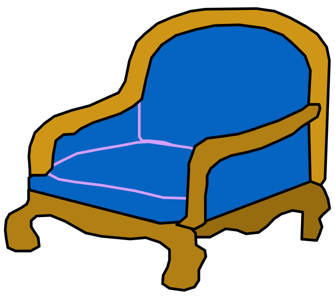
where $$\text{MAD} = \frac{\sum|\text{Actual} - \text{Forecast}|}{n}$$

Monitoring and controling forecast

- Using a tracking signal is a good way to make sure the forecasting system is continuing to do a good job

- even negative or positive

Choosing a Method
Forecast Error



Measures of Forecast Error

$$E_t = D_t - F_t$$

$$\text{CFE} = \Sigma E_t$$

$$\text{MSE} = \frac{\Sigma E_t^2}{n}$$

$$\text{MAD} = \frac{\Sigma |E_t|}{n}$$

$$\sigma = \sqrt{\frac{\Sigma (E_t - \bar{E})^2}{n - 1}}$$

$$\text{MAPE} = \frac{\Sigma\left[|E_t|\,(100)\right]/D_t}{n}$$

$$\text{Forecast error} = \text{Actual demand} - \text{Forecast value}$$
$$= A_t - F_t$$

Several measures are used in practice to calculate the overall forecast error. These measures can be used to compare different forecasting models, as well as to monitor forecasts to ensure they are performing well. Three of the most popular measures are mean absolute deviation (MAD), mean squared error (MSE), and mean absolute percent error (MAPE). We now describe and give an example of each.

Mean Absolute Deviation The first measure of the overall forecast error for a model is the mean absolute deviation (MAD). This value is computed by taking the sum of the absolute values of the individual forecast errors (deviations) and dividing by the number of periods of data (n):

$$\text{MAD} = \frac{\sum |\text{Actual} - \text{Forecast}|}{n} \tag{4-5}$$

FORECAST ERRORS MAD

MAPE

DETERMINING THE MEAN ABSOLUTE PERCENT ERROR (MAPE)

The Port of Baltimore wants to now calculate the MAPE when $\alpha = .10$.

APPROACH ▶ Equation (4-7) is applied to the forecast data computed in Example 4.

SOLUTION ▶

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST FOR $\alpha = .10$	ABSOLUTE PERCENT ERROR 100 (|ERROR|/ACTUAL)
1	180	175.00	100(5/180) = 2.78%
2	168	175.50	100(7.5/168) = 4.46%
3	159	174.75	100(15.75/159) = 9.90%
4	175	173.18	100(1.82/175) = 1.05%
5	190	173.36	100(16.64/190) = 8.76%
6	205	175.02	100(29.98/205) = 14.62%
7	180	178.02	100(1.98/180) = 1.10%
8	182	178.22	100(3.78/182) = 2.08%
			Sum of % errors = 44.75%

$$\text{MAPE} = \frac{\sum \text{absolute percent error}}{n} = \frac{44.75\%}{8} = 5.59\%$$

INSIGHT ▶ MAPE expresses the error as a percent of the actual values, undistorted by a single large value.

Monitoring and Controlling Forecasts

- **Tracking Signal**
- Measures **how well the forecast is predicting actual values**
- Ratio of cumulative forecast errors to mean absolute deviation (MAD)
  - Good tracking signal has low values
  - If forecasts are continually high or low, the forecast has a bias error

Monitoring and Controlling Forecasts (2 of 2)

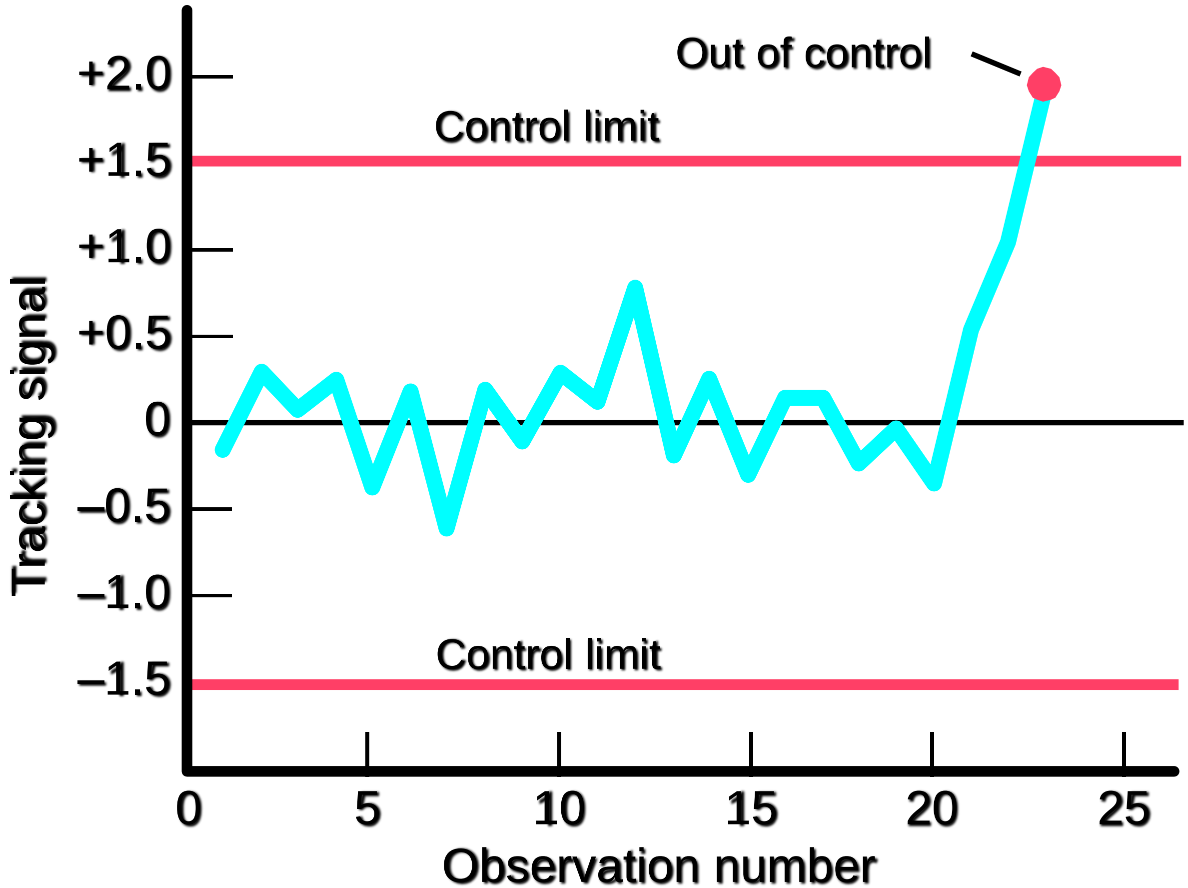
$$\text{Tracking signal} = \frac{\text{Cumulative error}}{\text{MAD}}$$

$$= \frac{\sum(\text{Actual demand in period i - Forecast demad in period i})}{\frac{\sum|\text{Actual} - \text{Forecast}|}{\text{n}}}$$

Choosing a Method

Tracking Signals

$$\text{Tracking signal} = \frac{\text{CFE}}{\text{MAD}}$$



Forecasting in the service sector

Forecasting at McDonald’s, FedEx, and Walmart is as important and complex as it is for manufacturers such as Toyota and Dell.

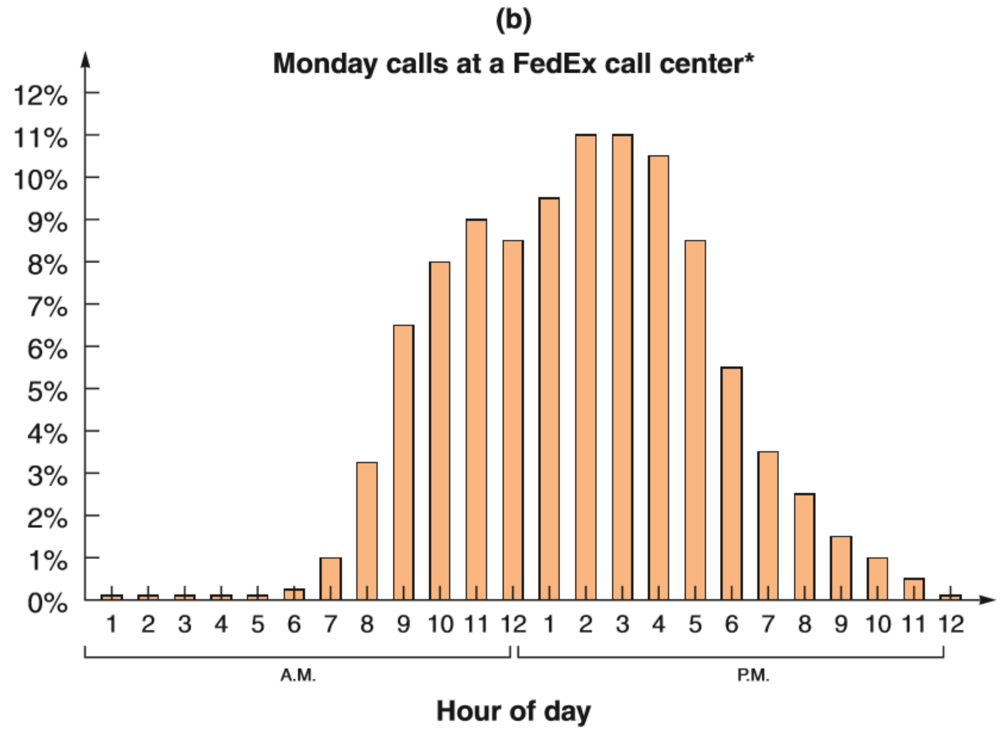
Specialty retail facilities, such as flower shops, may have other unusual demand patterns, and those patterns will differ depending on the holiday

Fast-food restaurants are well aware not only of weekly, daily, and hourly but even 15-minute variations in demands that influence sales. Therefore, detailed forecasts of demand are needed

Services again

- Taco Bell now use point-of-sale computers that track sales every quarter hour. Taco Bell found that a 6-week moving average was the forecasting technique that minimized its mean squared error (MSE) of these quarter-hour forecasts.
- Projections of customer transactions. These in turn are used by store managers to schedule staff, who begin in 15-minute increments, not 1-hour blocks as in other industries. The forecasting model has been so successful that Taco Bell has increased customer service while documenting more than $50 million in labor cost savings in 4 years of use.

(a)
Hourly sales at a fast-food restaurant
Percent of sales by hour of day
20%
15%
10%
5%
11–12
12–1
1–2
2–3
3–4
4–5
5–6
6–7
7–8
8–9
9–10
10–11
(Lunchtime)
(Dinnertime)
Hour of day



(b)
Monday calls at a FedEx call center*
12%
11%
10%
9%
8%
7%
6%
5%
4%
3%
2%
1%
0%
1 2 3 4 5 6 7 8 9 10 11 12 1 2 3 4 5 6 7 8 9 10 11 12
A.M.
P.M.
Hour of day

TEST

1- Forecasting time horizons include :

a) long range.

b) medium range.

c) short range.

d) all of the above.

2- Average demand for iPods in the Rome, Italy, Apple store is 800 units per month. The May monthly index is 1.25. What is the seasonally adjusted sales forecast for May?

a) 640 units

b) 798.75 units

c) 800 units

d) 1,000 units

e) cannot be calculated with the information given

To continue

3- Qualitative methods of forecasting include:

a) sales force composite.

b) jury of executive opinion.

c) consumer market survey.

d) exponential smoothing.

e) all except d

4- The tracking signal is the:

a) standard error of the estimate.

b) cumulative error.

c) mean absolute deviation (MAD).

d) ratio of the cumulative error to MAD.

e) mean absolute percent error (MAPE).

Self test

5-The difference between a *moving-average* model and an

exponential smoothing model is that ……

6-Three popular measures of forecast accuracy are:

a) total error, average error, and mean error.

b) average error, median error, and maximum error.

c) median error, minimum error, and maximum absolute error.

d) mean absolute deviation, mean squared error, and mean absolute percent error.

Application

COMPUTING THE TRACKING SIGNAL AT CARLSON'S BAKERY

Carlson's Bakery wants to evaluate performance of its croissant forecast.

APPROACH ▶ Develop a tracking signal for the forecast, and see if it stays within acceptable limits, which we define as ± 4 MADs.

SOLUTION ▶ Using the forecast and demand data for the past 6 quarters for croissant sales, we develop a tracking signal in the following table:

QUARTER	ACTUAL DEMAND	FORECAST DEMAND	ERROR	CUMULATIVE ERROR	ABSOLUTE FORECAST ERROR	CUMULATIVE ABSOLUTE FORECAST ERROR	MAD	TRACKING SIGNAL (CUMULATIVE ERROR/MAD)
1	90	100	−10	−10	10	10	10.0	−10/10 = −1
2	95	100	−5	−15	5	15	7.5	−15/7.5 = −2
3	115	100	+15	0	15	30	10.0	0/10 = 0
4	100	110	−10	−10	10	40	10.0	−10/10 = −1
5	125	110	+15	+5	15	55	11.0	+5/11 = +0.5
6	140	110	+30	+35	30	85	14.2	+35/14.2 = +2.5

$$\text{At the end of quarter 6, MAD} = \frac{\sum|\text{Forecast errors}|}{n} = \frac{85}{6} = 14.2$$

$$\text{and Tracking signal} = \frac{\text{Cumulative error}}{\text{MAD}} = \frac{35}{14.2} = 2.5 \text{ MADs}$$

Exercice

The following data come from regression line projections:

PERIOD	FORECAST VALUES	ACTUAL VALUES
1	410	406
2	419	423
3	428	423
4	435	440

Compute the MAD and MSE.

Another

Room registrations in the Toronto Towers Plaza Hotel have been recorded for the past 9 years. To project future occupancy, management would like to determine the mathematical trend of guest registration. This estimate will help the hotel determine whether future expansion will be needed. Given the following time-series data, develop a regression equation relating registrations to time (e.g., a trend equation). Then forecast year 11 registrations. Room registrations are in the thousands:

Year 1: 17	Year 2: 16	Year 3: 16	Year 4: 21	Year 5: 20
Year 6: 20	Year 7: 23	Year 8: 25	Year 9: 24	

Seasonal

Quarterly demand for Ford F150 pickups at a New York auto dealer is forecast with the equation:

$$\hat{y} = 10 + 3x$$

where x = quarters, and:

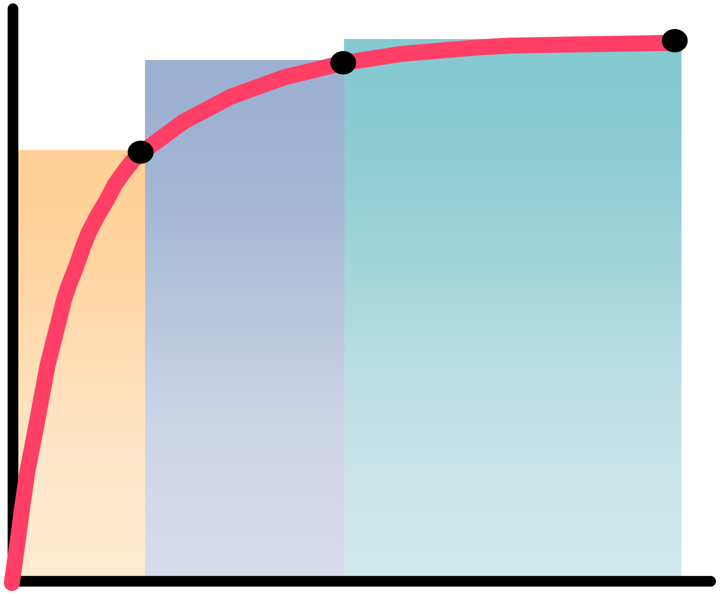
Quarter I of year 1 = 0
Quarter II of year 1 = 1
Quarter III of year 1 = 2
Quarter IV of year 1 = 3
Quarter I of year 2 = 4
and so on

and:

$$\hat{y} = \text{quarterly demand}$$

The demand for trucks is seasonal, and the indices for Quarters I, II, III, and IV are 0.80, 1.00, 1.30, and 0.90, respectively. Forecast demand for each quarter of year 3. Then, seasonalize each forecast to adjust for quarterly variations.

Inventory Management



Inventory management why ?

- When Jeff Bezos opened his revolutionary business in 1995, **Amazon.com** was intended to be a "virtual" retailer—no inventory, **no warehouses, no overhead**—just a bunch of computers taking orders for books and authorizing others to fill them.
- Now, **Amazon stocks millions of items of inventory**, amid hundreds of thousands of bins on shelves in over 150 warehouses around the world.

Why inventory ?

1. To provide a selection of goods **for anticipated customer demand** and to separate the firm from fluctuations in that demand. Such inventories are typical in retail establishments.

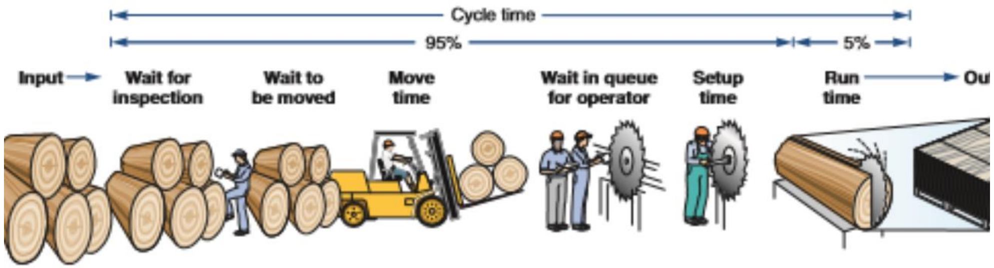
2. To **decouple various parts of the production process**. For example, if a firm's supplies fluctuate, extra inventory may be necessary to decouple the production process from suppliers.

3. To take **advantage of quantity discounts**, because purchases in larger quantities may reduce the cost of goods or their delivery.

4. To hedge **against** inflation and upward price **changes**

Types of inventory

- **Raw material inventory** Materials that are usually purchased but have yet to enter the manufacturing process.
- **Work-in-process (WIP)** inventory Products or components that are no longer raw materials but have yet to become finished products.
- **Maintenance/repair/operating (MRO) inventory** Maintenance, repair, and operating materials.



ure 12.1

Material Flow Cycle

st of the time that work is in-process (95% of the cycle time) is not productive time.

Inventory Costs

- Interest or Opportunity Cost
- Storage and Handling Costs
- Taxes, Insurance, and Shrinkage

Inventory Costs

- Customer Service
- Ordering Cost
- Setup Cost
- Labor and
  Equipment Utilization
- Transportation Costs
- Payments to Suppliers

Types of Inventory

Cycle Inventory

$$\text{Average cycle inventory} = \frac{Q + 0}{2}$$

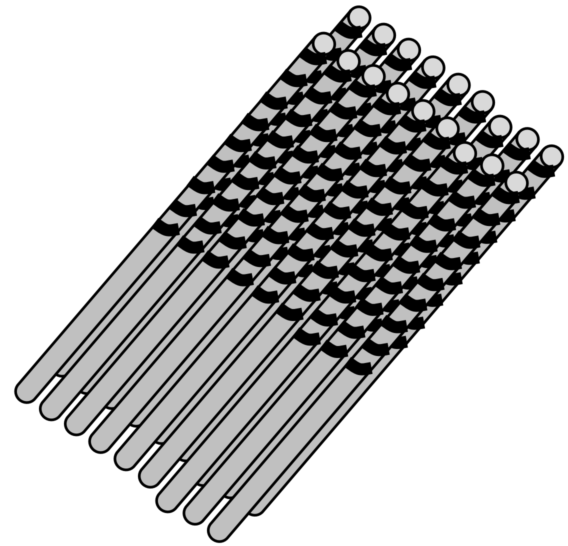
Safety Stock Inventory

Anticipation Inventory

Pipeline Inventory

$$\text{Pipeline inventory} = \overline{D}_L = dL$$

Types of Inventory

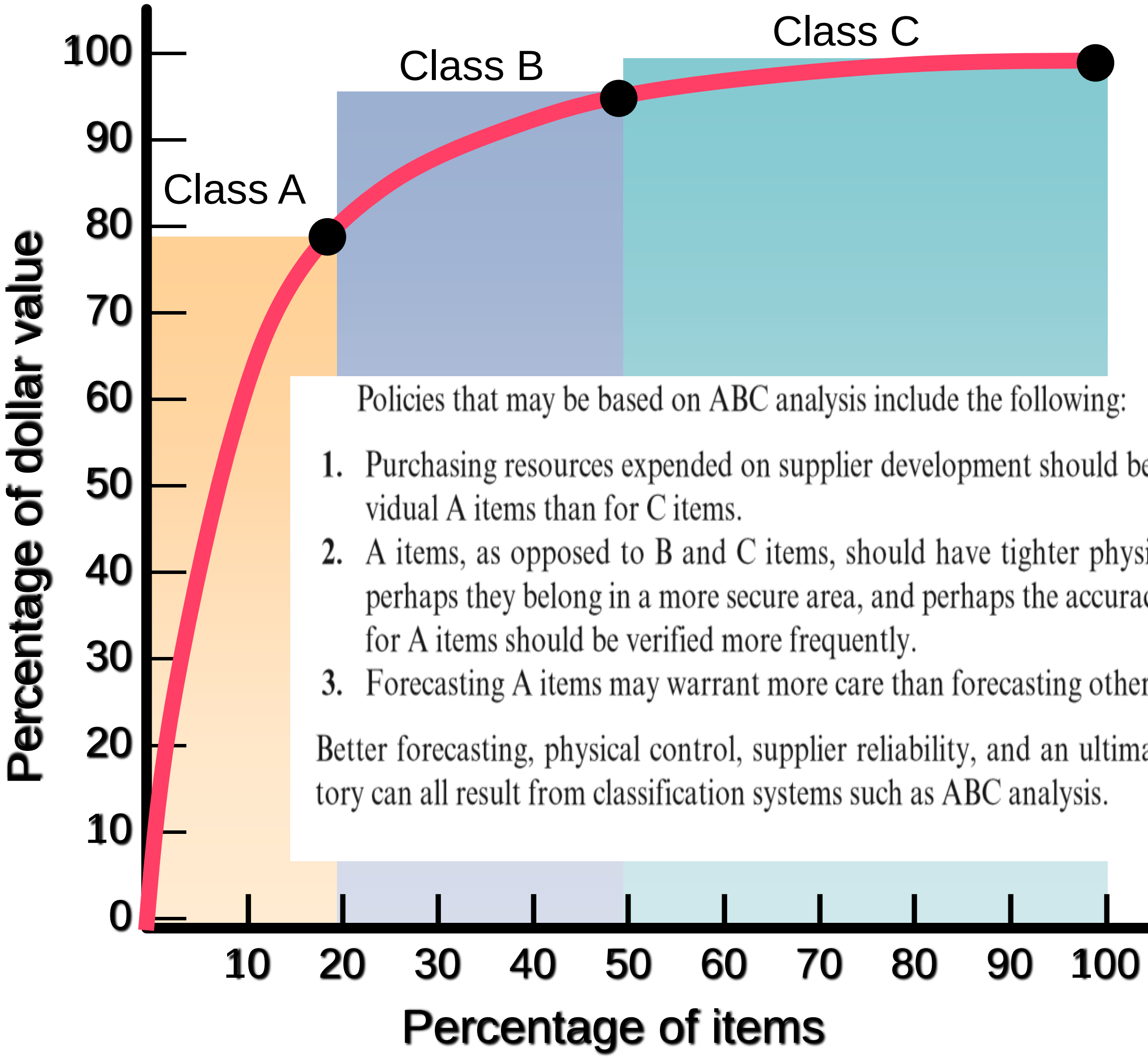


Cycle inventory $= Q/2$
$= 280/2$
$= 140$ drills

Pipeline inventory $= \bar{D}_L = dL$
$=$ (70 drills/week)(3 weeks)
$= 210$ drills

Example 15.1

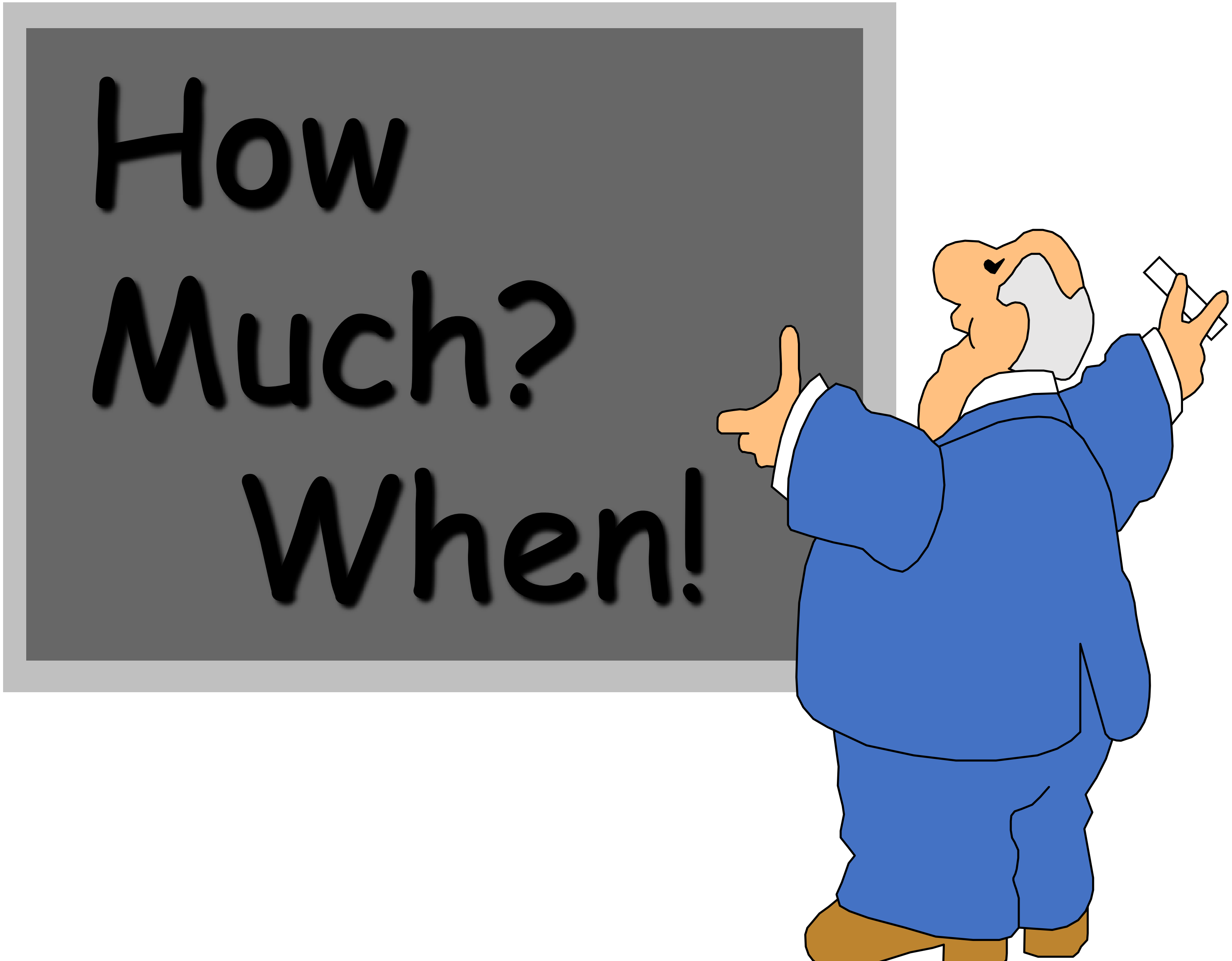
ABC Analysis



Policies that may be based on ABC analysis include the following:

1. Purchasing resources expended on supplier development should be much higher for individual A items than for C items.
2. A items, as opposed to B and C items, should have tighter physical inventory control; perhaps they belong in a more secure area, and perhaps the accuracy of inventory records for A items should be verified more frequently.
3. Forecasting A items may warrant more care than forecasting other items.

Better forecasting, physical control, supplier reliability, and an ultimate reduction in inventory can all result from classification systems such as ABC analysis.


How
Much?
When!

Record accuracy

And Cycle counting

- A continuing reconciliation of inventory with inventory records

Omnicell

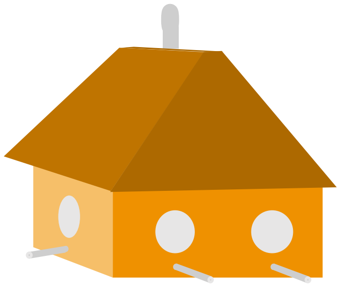
In this hospital, these vertically rotating storage carousels provide rapid access to hundreds of critical items and at the same tim save floor space. This Omnicell inventory management carousel is also secure and has the added advantage of printing bar code labels.

Cycle counting example (pipeline)

- CYCLE COUNTING AT COLE'S TRUCKS, INC. Cole's Trucks, Inc., a builder of high-quality refuse trucks, has about 5,000 items in its inventory. It wants to determine how many items to cycle count each day.
- APPROACH After hiring Matt Clark, a bright young OM student, for the summer, the firm determined that it has
  - 500 A items, 1,750 B items, and 2,750 C items.
- Company policy is to count all A items every month (every 20 working days), all B items every quarter (every 60 working days), and all C items every 6 months (every 120 working days). The firm then allocates some items to be counted each day.

Economic Order Quantity

Economic Order Quantity



Assumptions

1. Demand rate is constant
2. No constraints on lot size
3. Only relevant costs are holding and ordering/setup
4. Decisions for items are independent from other items
5. No uncertainty in lead time or supply

Economic Order Quantity

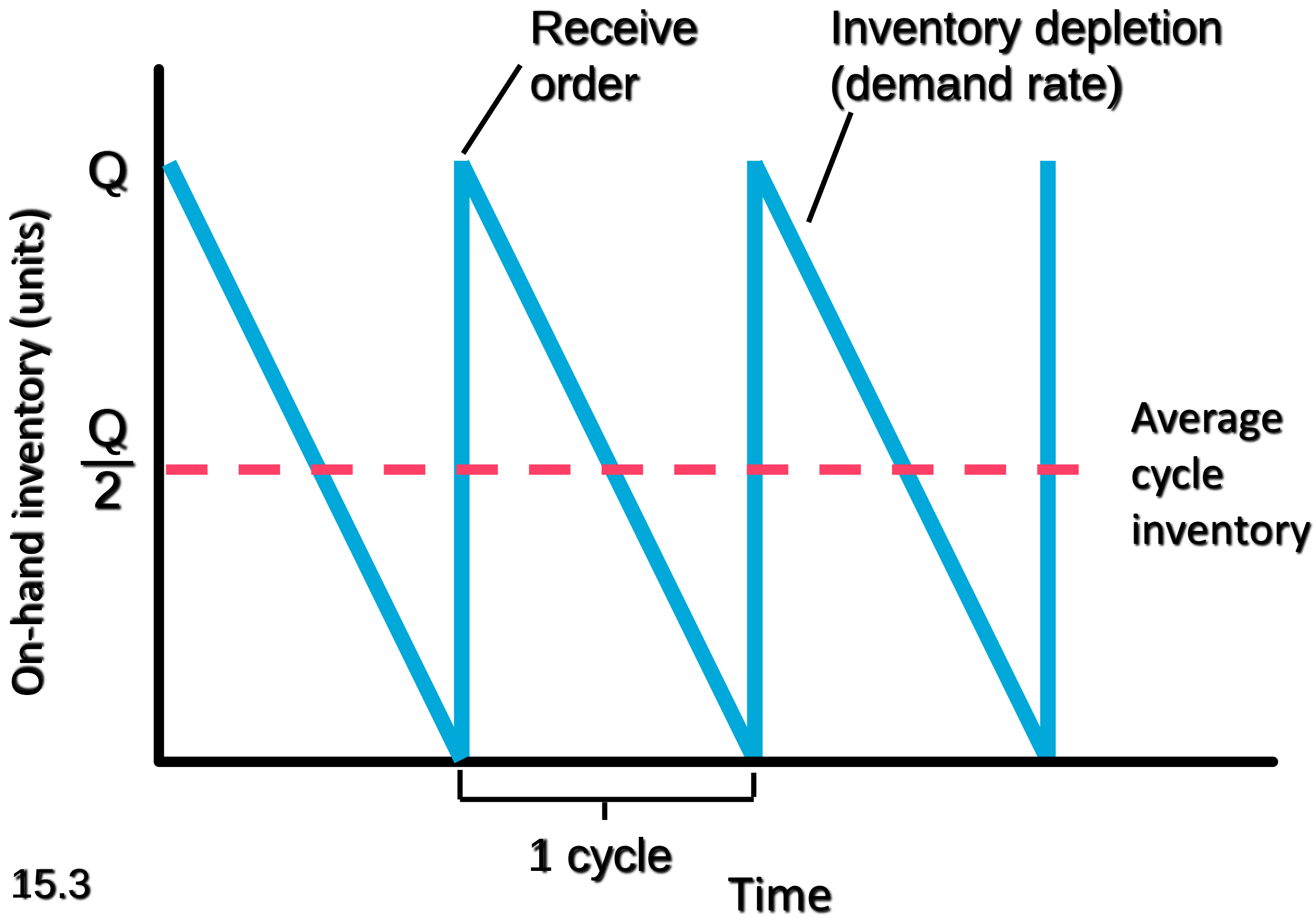


Figure 15.3

Economic Order Quantity

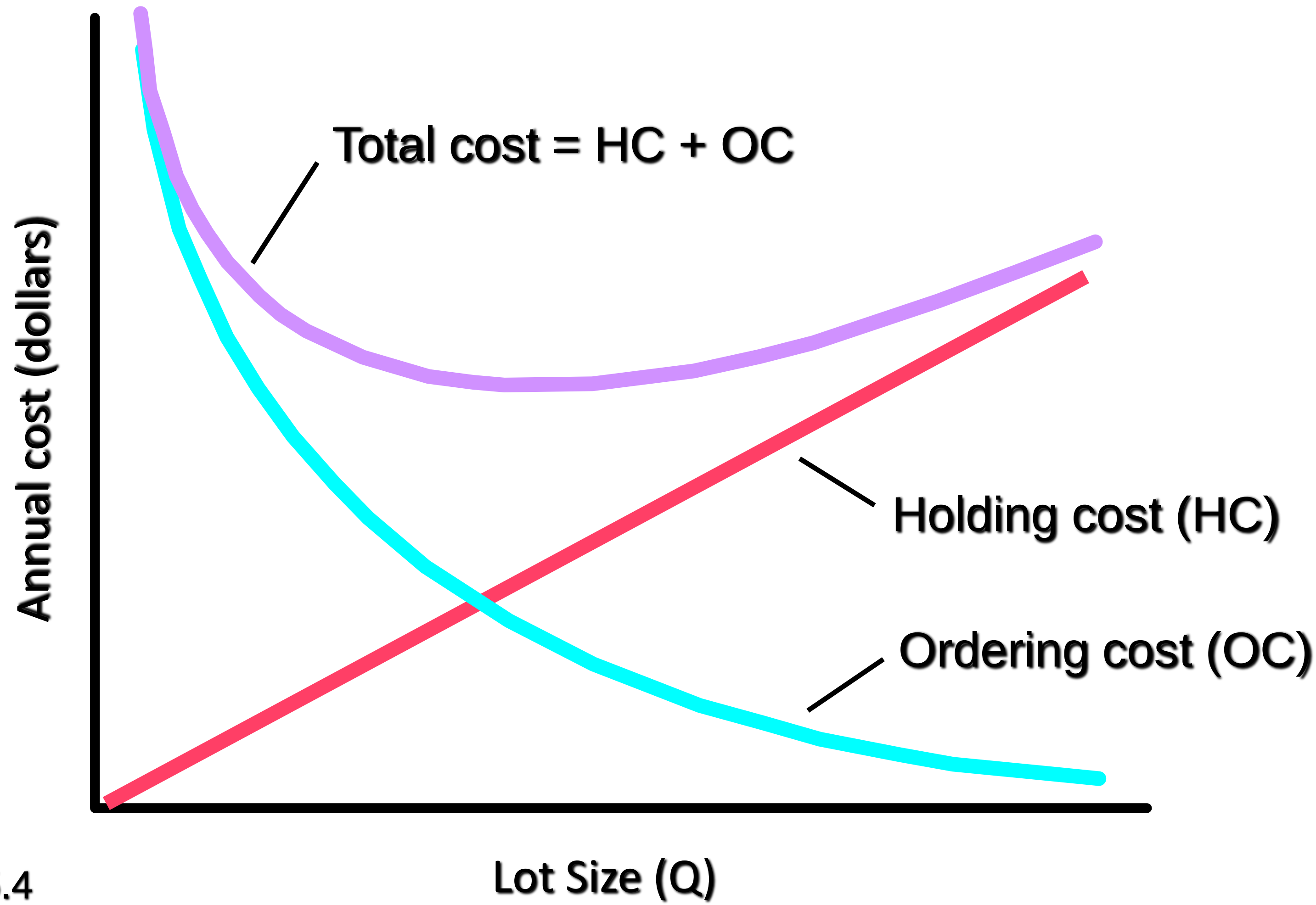


Figure 15.4

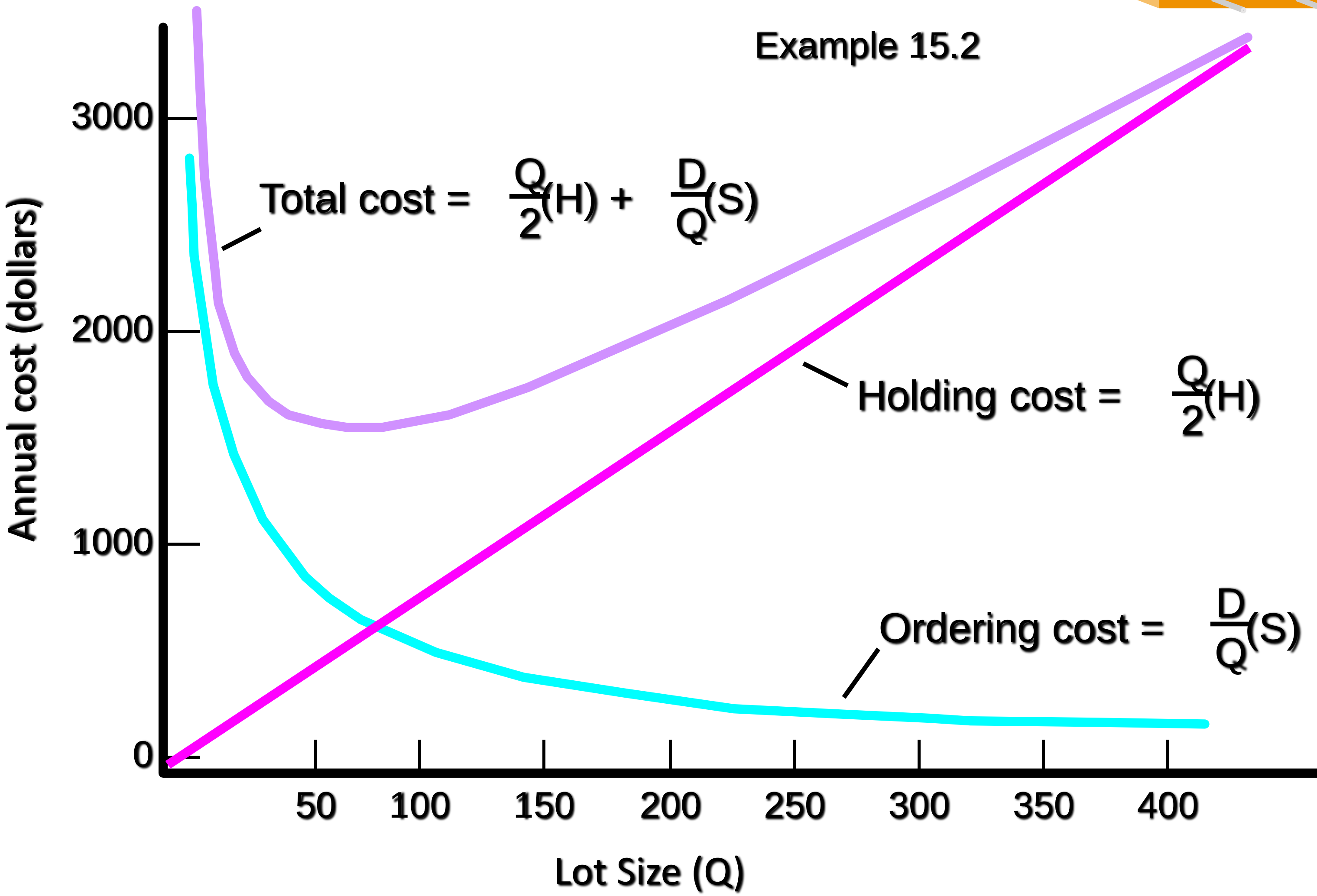
EXAMPLE

Total cost is Holding cost + ordering cost

- Costing **Out a Lot-Sizing Policy**
- A museum of natural history opened a gift shop two years ago. Managing inventories has become a problem. Low inventory turnover is squeezing profit margins and causing cash-flow *problems.*
- One of the top-selling items in the container group at the museum's gift shop is a bird feeder. Sales are 18 units per week, and the supplier charges $60 per unit. The cost of placing an order with the supplier is $45. Annual holding cost is 25 percent of a feeder's value, and the museum operates 52 weeks per year. Management chose a 390-unit lot size so that new orders could be placed less frequently. What is the annual cost of the current policy of using a 390-unit lot size? Would a lot size of 468 be better?
-

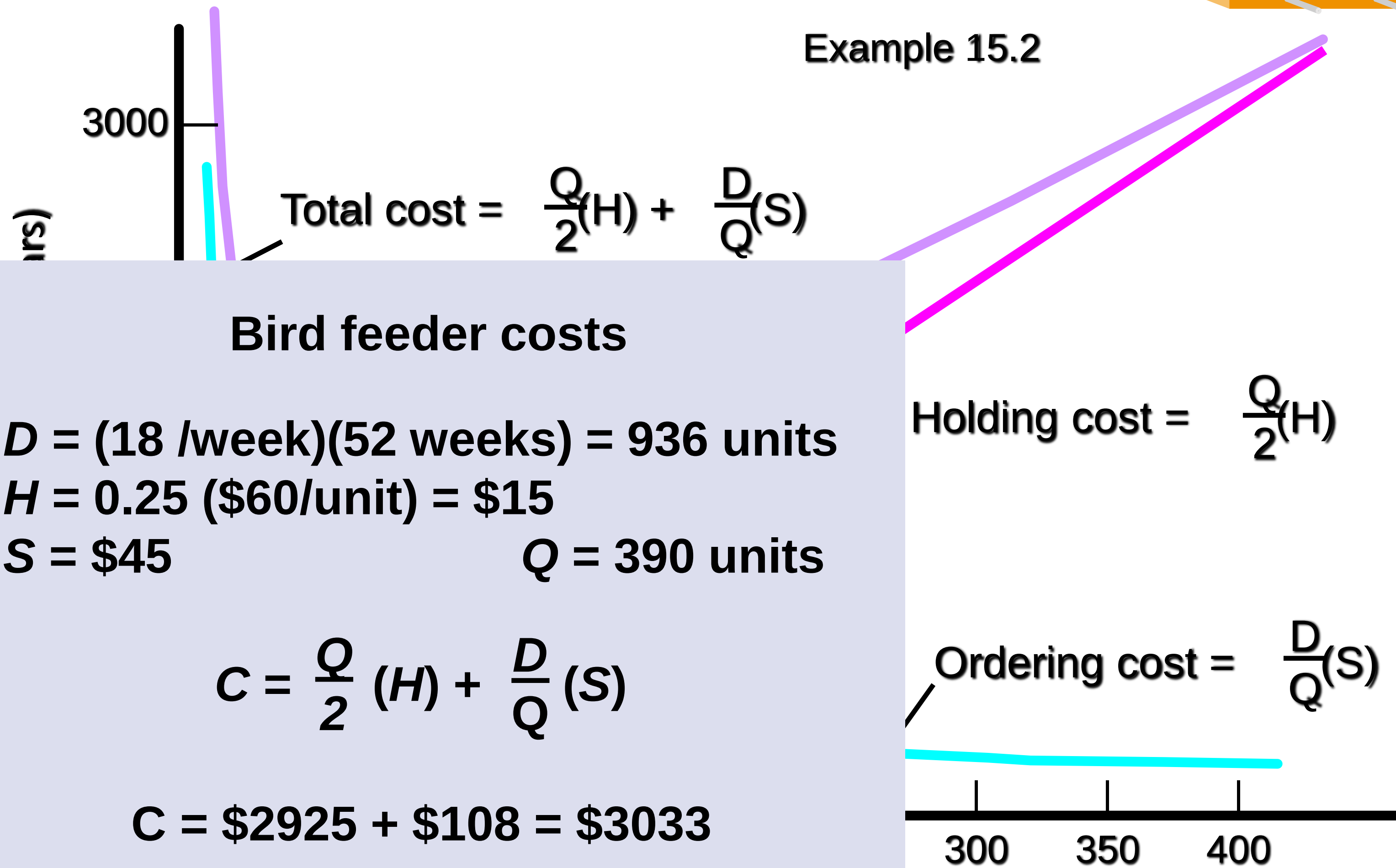
Economic Order Quantity

Example 15.2



Economic Order Quantity

Example 15.2



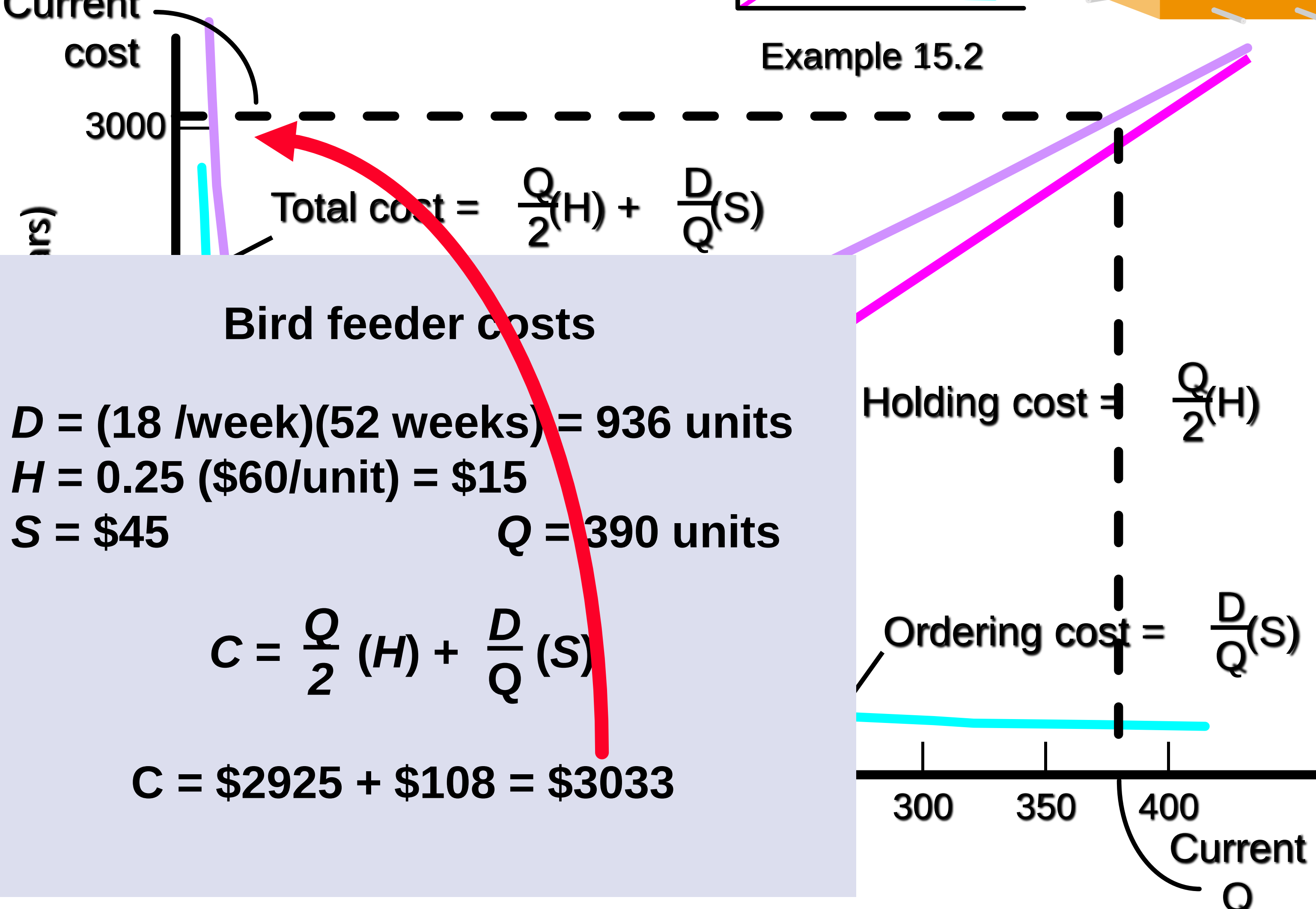
Bird feeder costs

D **= (18 /week)(52 weeks) = 936 units**
H **= 0.25 ($60/unit) = $15**
S **= $45** ***Q*** **= 390 units**

$$C = \frac{Q}{2}(H) + \frac{D}{Q}(S)$$

C = $2925 + $108 = $3033

Economic Order Quantity



Bird feeder costs

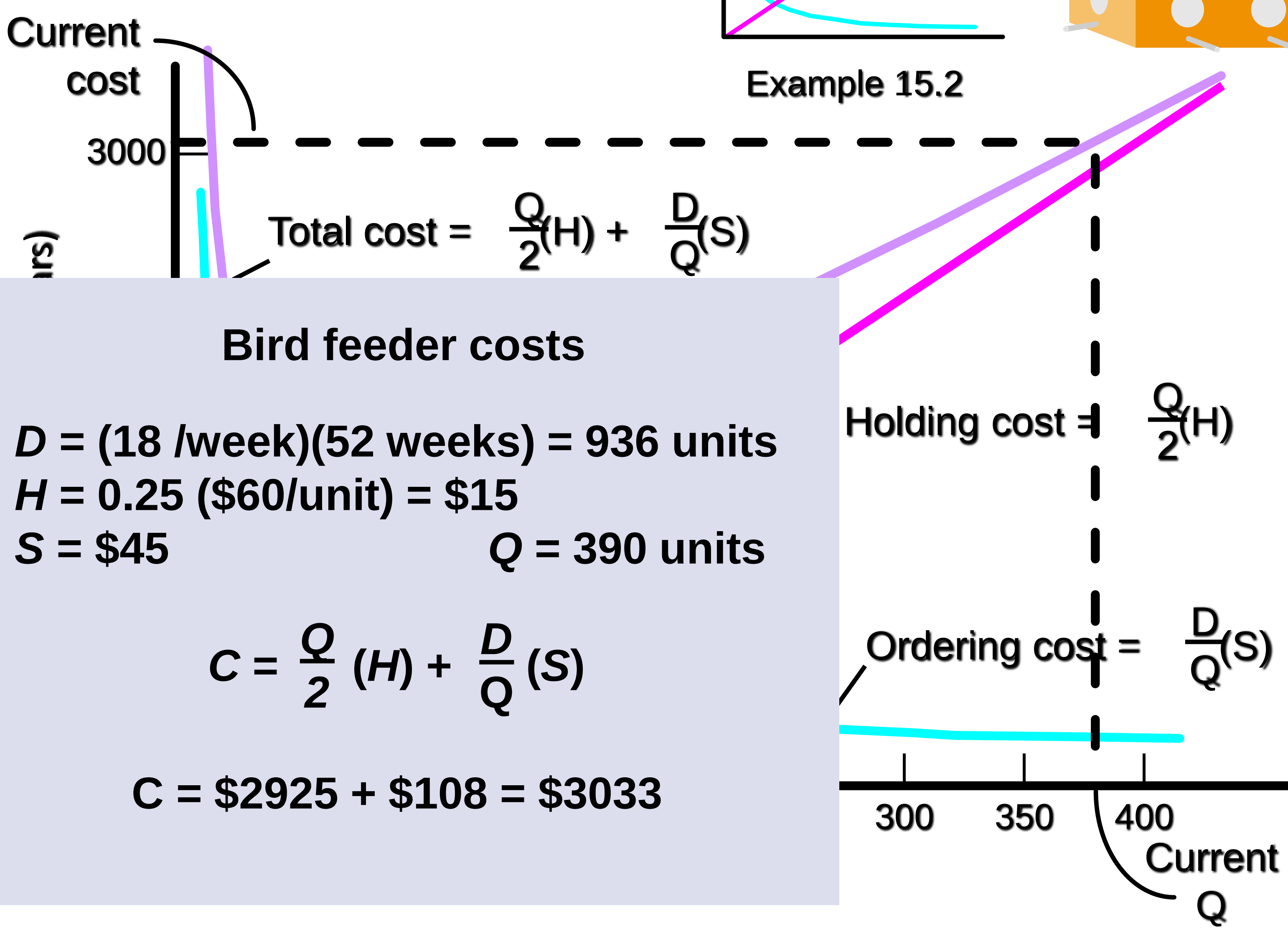
D **= (18 /week)(52 weeks) = 936 units**
H **= 0.25 ($60/unit) = $15**
S **= $45** ***Q*** **= 390 units**

$$C = \frac{Q}{2}(H) + \frac{D}{Q}(S)$$

C = $2925 + $108 = $3033

Economic Order Quantity

Example 15.2



Bird feeder costs

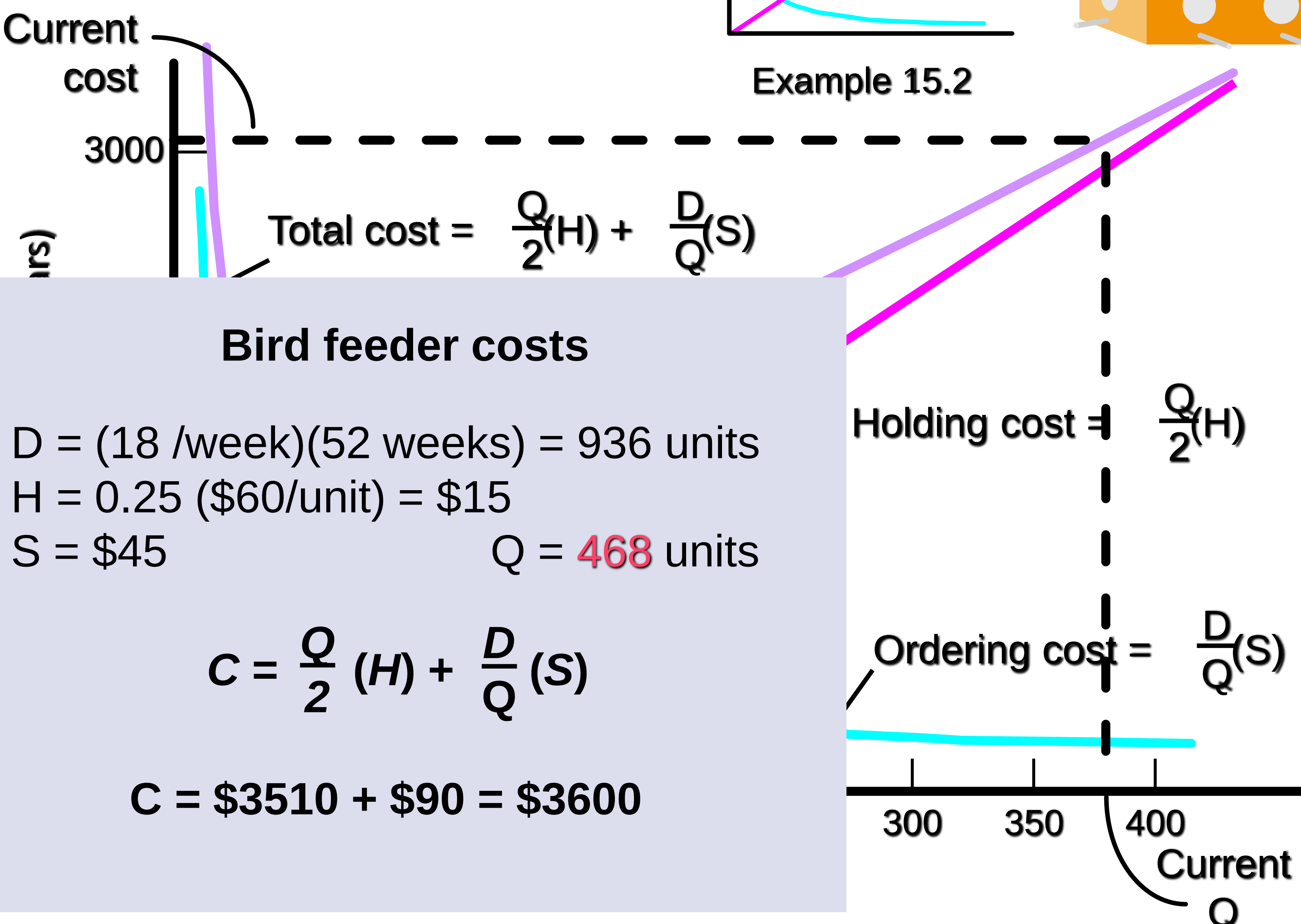
D **= (18 /week)(52 weeks) = 936 units**
H **= 0.25 ($60/unit) = $15**
S **= $45** ***Q*** **= 390 units**

$$C = \frac{Q}{2}(H) + \frac{D}{Q}(S)$$

C = $2925 + $108 = $3033

Economic Order Quantity

Example 15.2



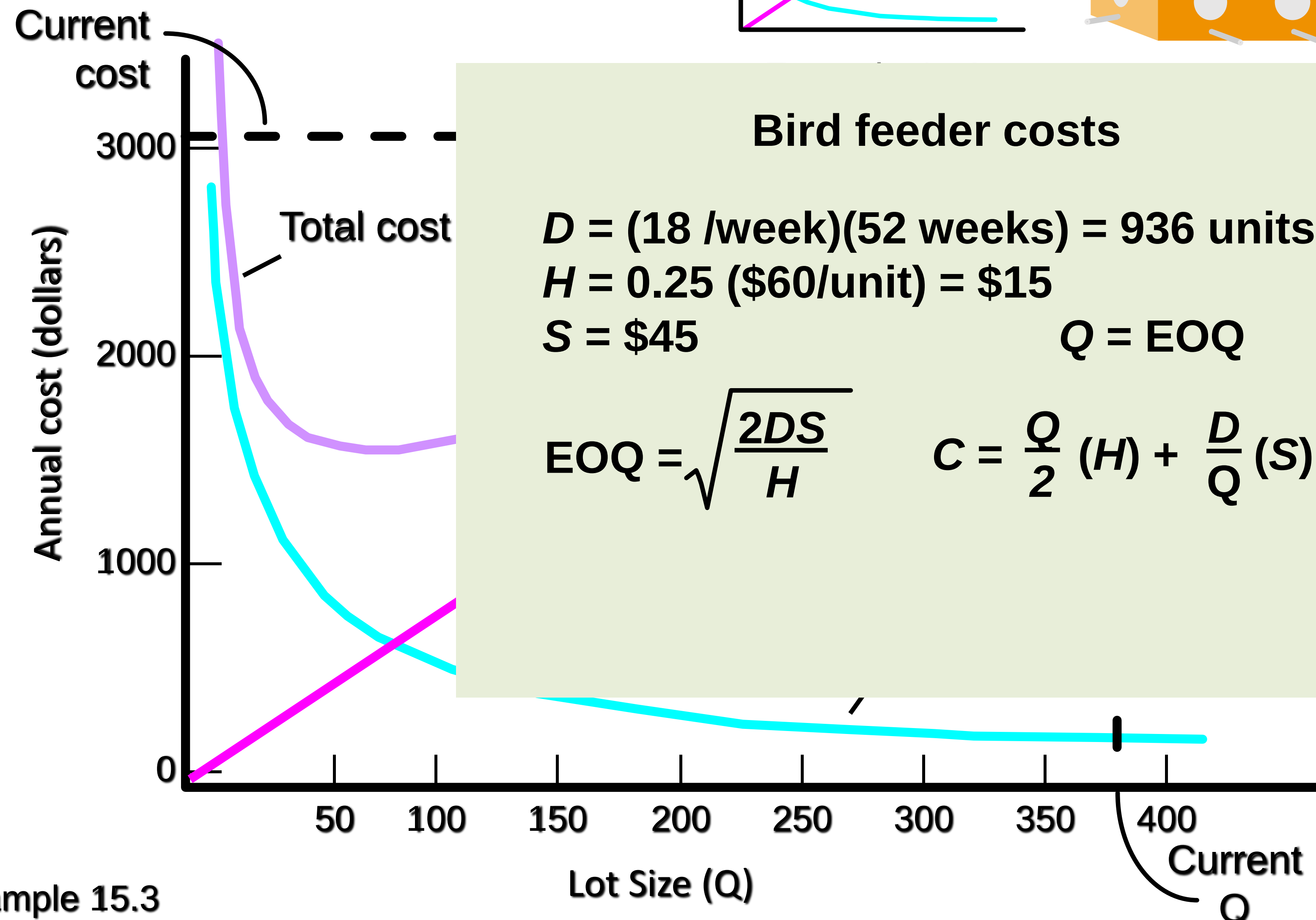
Bird feeder costs

D = (18 /week)(52 weeks) = 936 units
H = 0.25 ($60/unit) = $15
S = $45 Q = 468 units

$$C = \frac{Q}{2}(H) + \frac{D}{Q}(S)$$

C = $3510 + $90 = $3600

Economic Order Quantity



Bird feeder costs

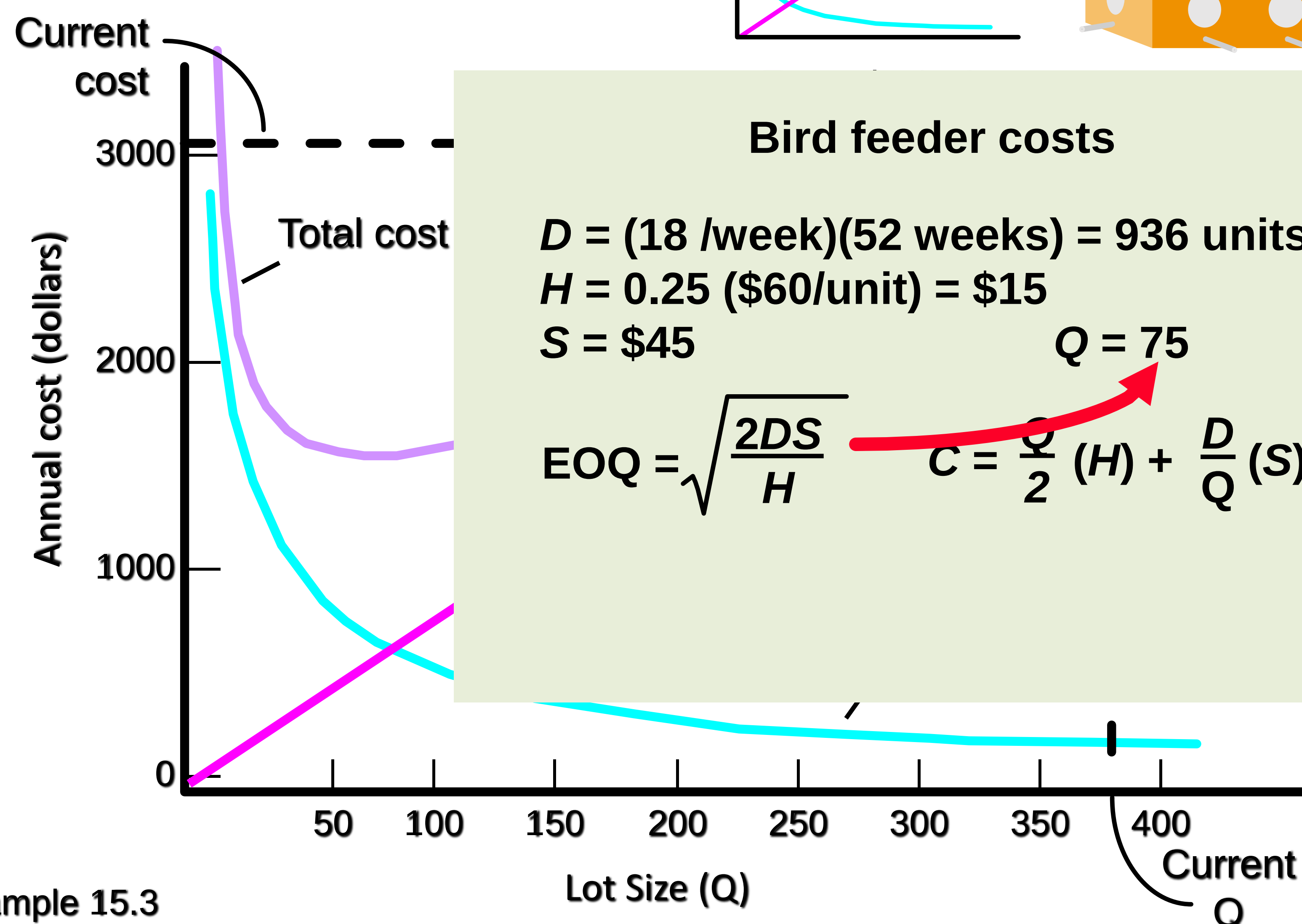
D = (18 /week)(52 weeks) = 936 units
H = 0.25 ($60/unit) = $15
S = $45

Q = EOQ

$$\text{EOQ} = \sqrt{\frac{2DS}{H}} \qquad C = \frac{Q}{2}(H) + \frac{D}{Q}(S)$$

Example 15.3

Economic Order Quantity



Bird feeder costs

D = (18 /week)(52 weeks) = 936 units

H = 0.25 ($60/unit) = $15

S = $45

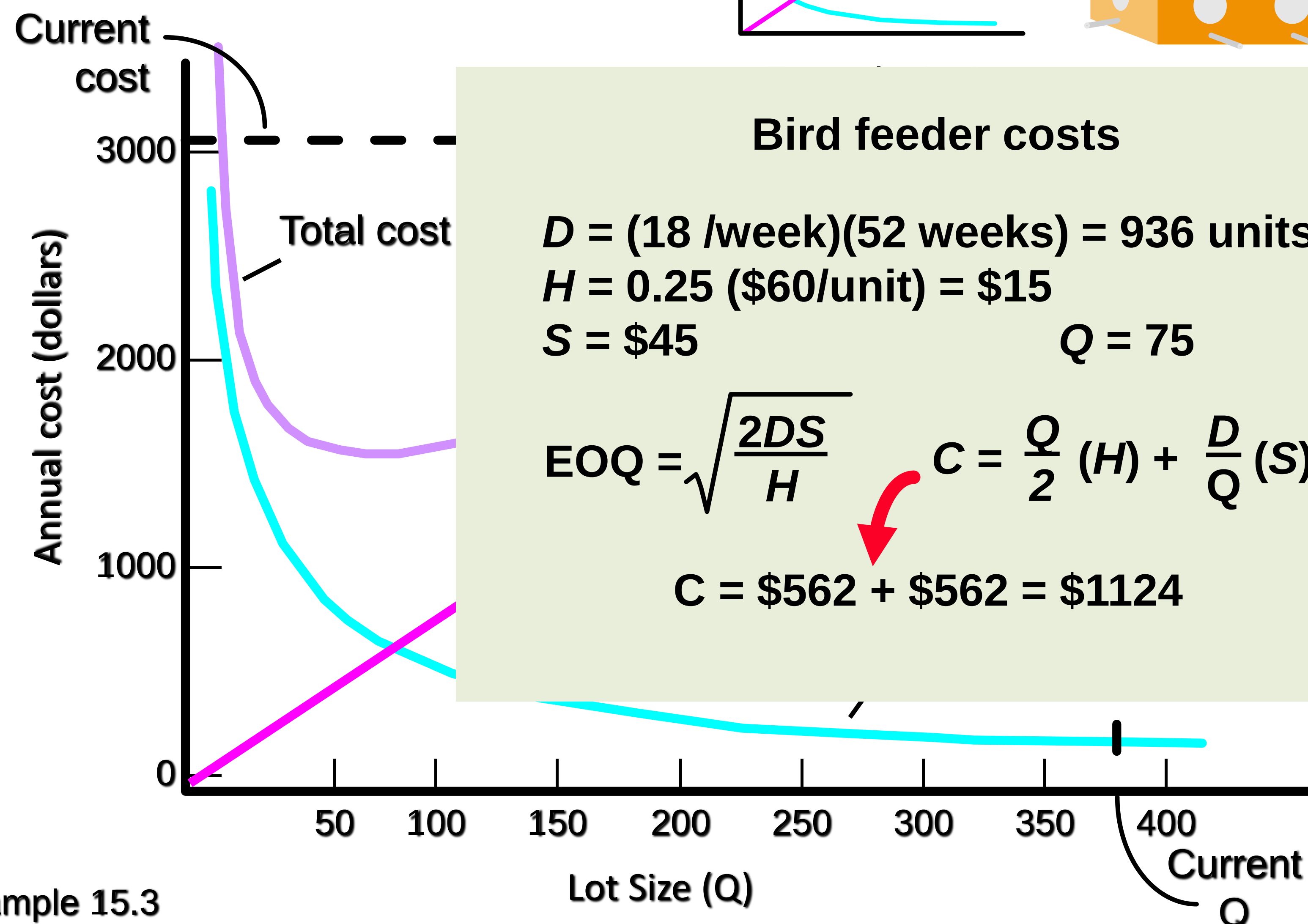
Q = 75

$$\text{EOQ} = \sqrt{\frac{2DS}{H}}$$

$$C = \frac{Q}{2}(H) + \frac{D}{Q}(S)$$

Example 15.3

Economic Order Quantity



Bird feeder costs

D = (18 /week)(52 weeks) = 936 units

H = 0.25 ($60/unit) = $15

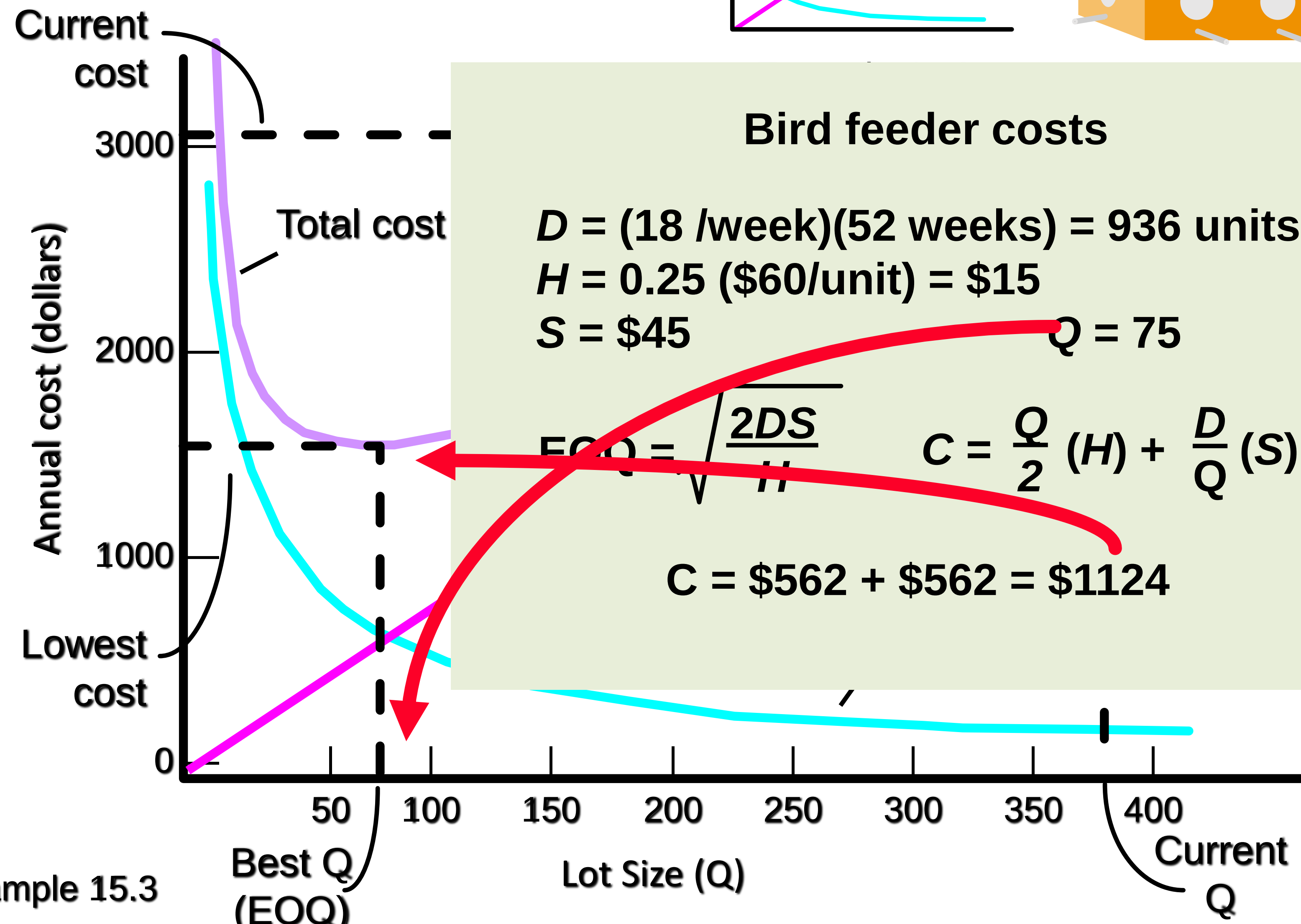
S = $45 $\qquad$ Q = 75

$$EOQ = \sqrt{\frac{2DS}{H}} \qquad C = \frac{Q}{2}(H) + \frac{D}{Q}(S)$$

C = $562 + $562 = $1124

Example 15.3

Economic Order Quantity



Bird feeder costs

$D = (18\text{ /week})(52\text{ weeks}) = 936\text{ units}$

$H = 0.25(\$60/\text{unit}) = \$15$

$S = \$45$

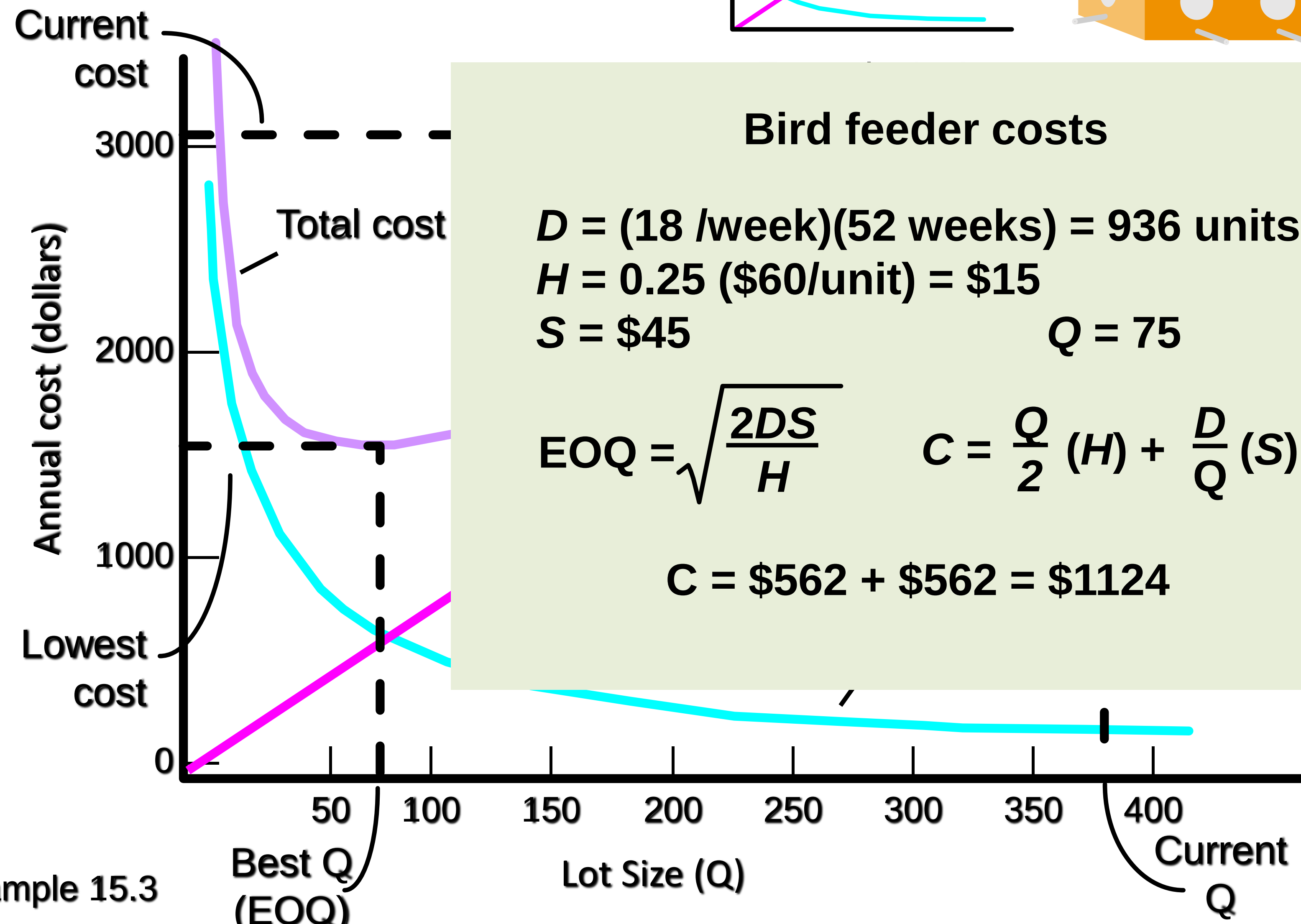
$Q = 75$

$$EOQ = \sqrt{\frac{2DS}{H}} \qquad C = \frac{Q}{2}(H) + \frac{D}{Q}(S)$$

C = \$562 + \$562 = \$1124

Example 15.3

Economic Order Quantity



Bird feeder costs

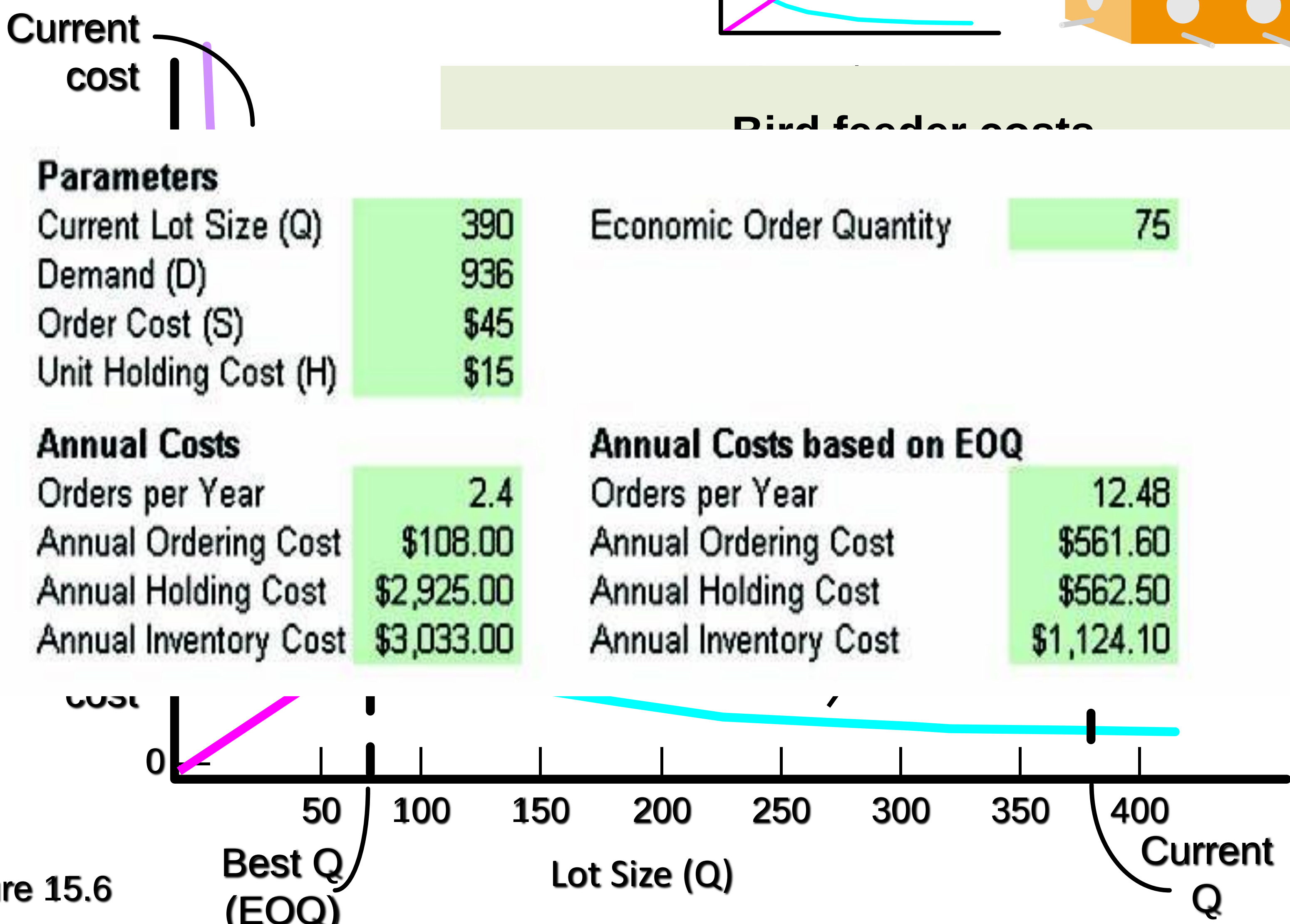
D = (18 /week)(52 weeks) = 936 units
H = 0.25 ($60/unit) = $15
S = $45 **$Q$ = 75**

$$\text{EOQ} = \sqrt{\frac{2DS}{H}} \qquad C = \frac{Q}{2}(H) + \frac{D}{Q}(S)$$

C = $562 + $562 = $1124

Example 15.3

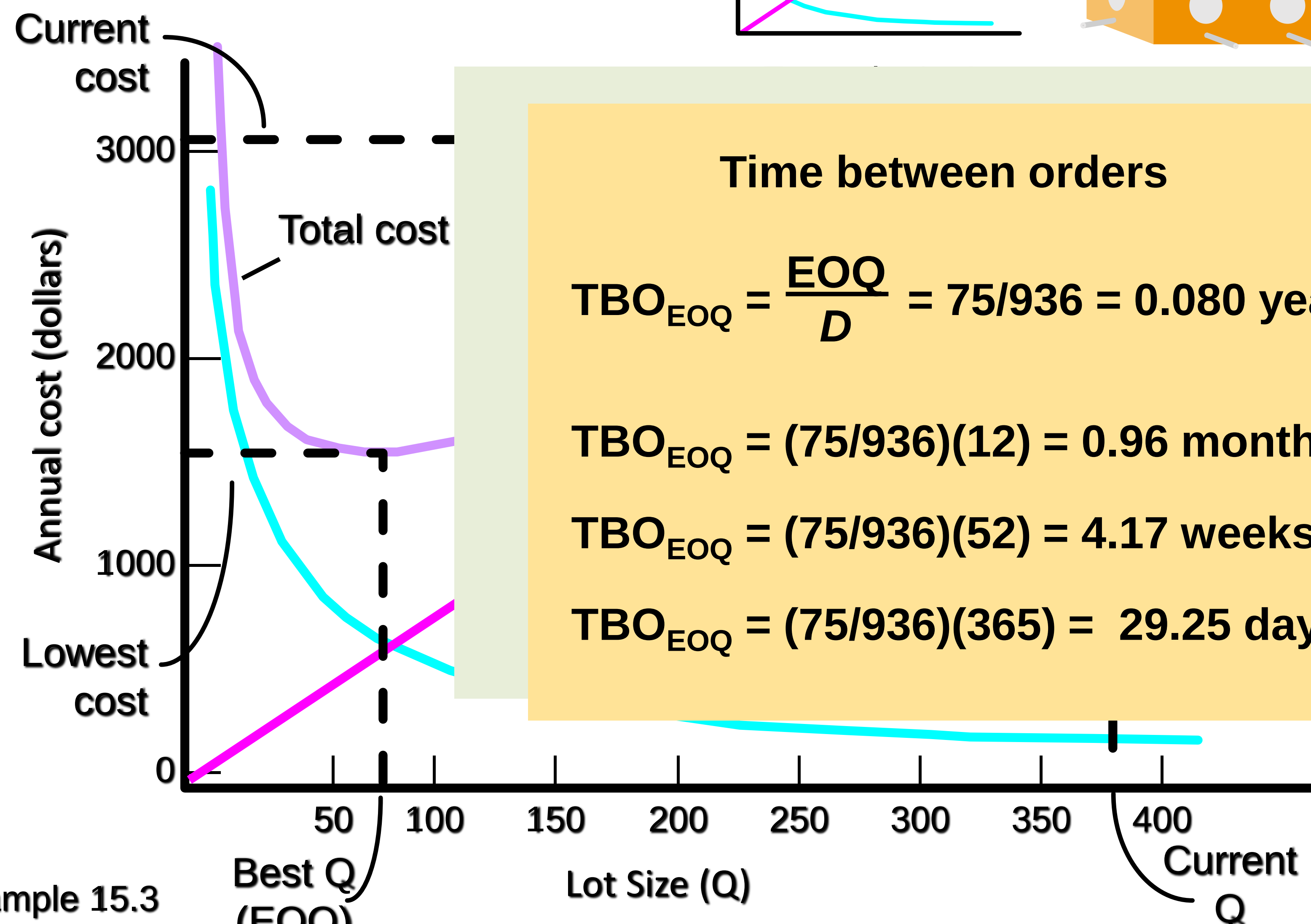
Economic Order Quantity



Parameters			
Current Lot Size (Q)	390	Economic Order Quantity	75
Demand (D)	936		
Order Cost (S)	$45		
Unit Holding Cost (H)	$15		
Annual Costs		**Annual Costs based on EOQ**	
Orders per Year	2.4	Orders per Year	12.48
Annual Ordering Cost	$108.00	Annual Ordering Cost	$561.60
Annual Holding Cost	$2,925.00	Annual Holding Cost	$562.50
Annual Inventory Cost	$3,033.00	Annual Inventory Cost	$1,124.10

Figure 15.6

Economic Order Quantity



Time between orders

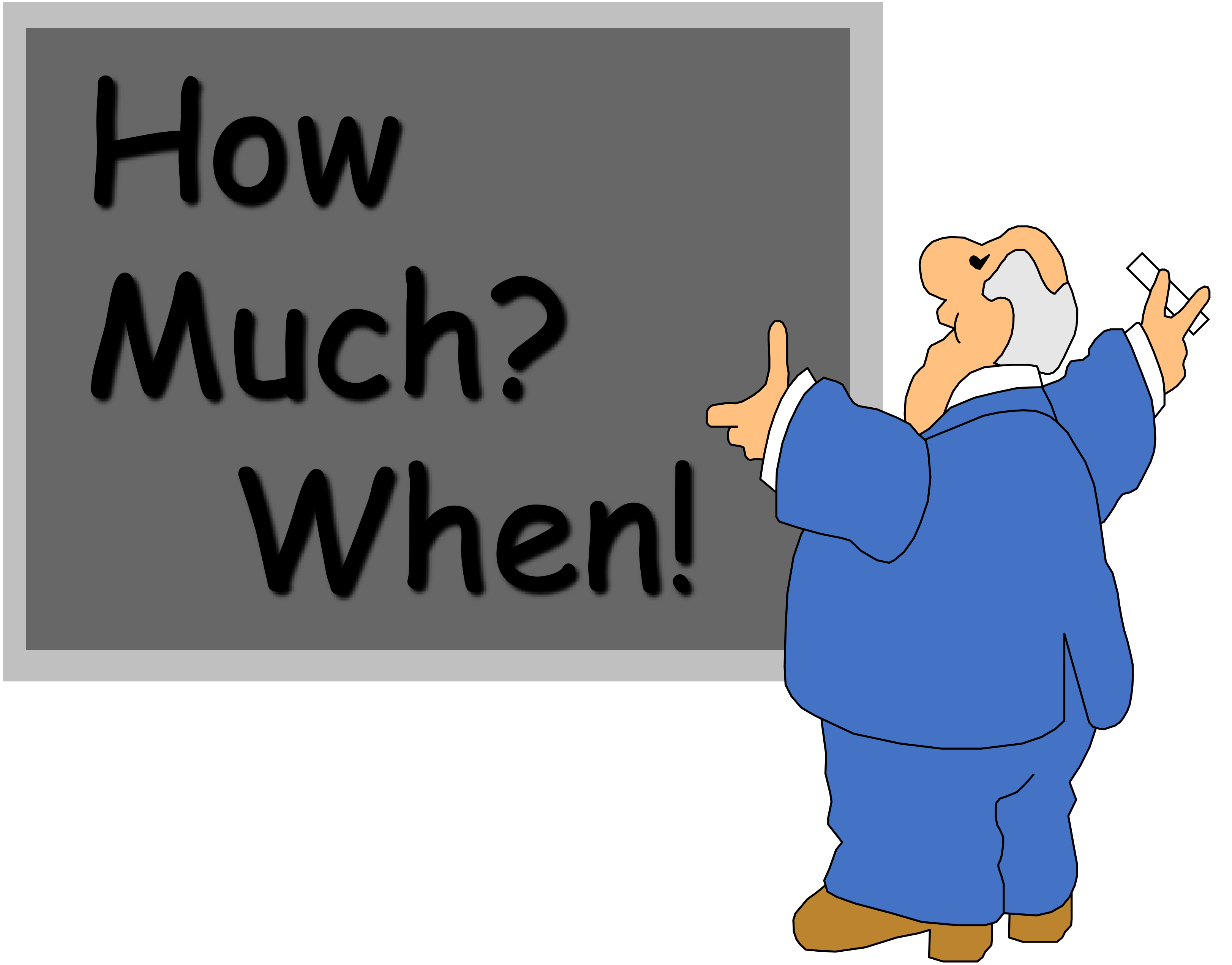
$$TBO_{EOQ} = \frac{EOQ}{D} = 75/936 = 0.080 \text{ year}$$

$$TBO_{EOQ} = (75/936)(12) = 0.96 \text{ months}$$

$$TBO_{EOQ} = (75/936)(52) = 4.17 \text{ weeks}$$

$$TBO_{EOQ} = (75/936)(365) = 29.25 \text{ days}$$

Example 15.3


How
Much? When!

Continuous Review

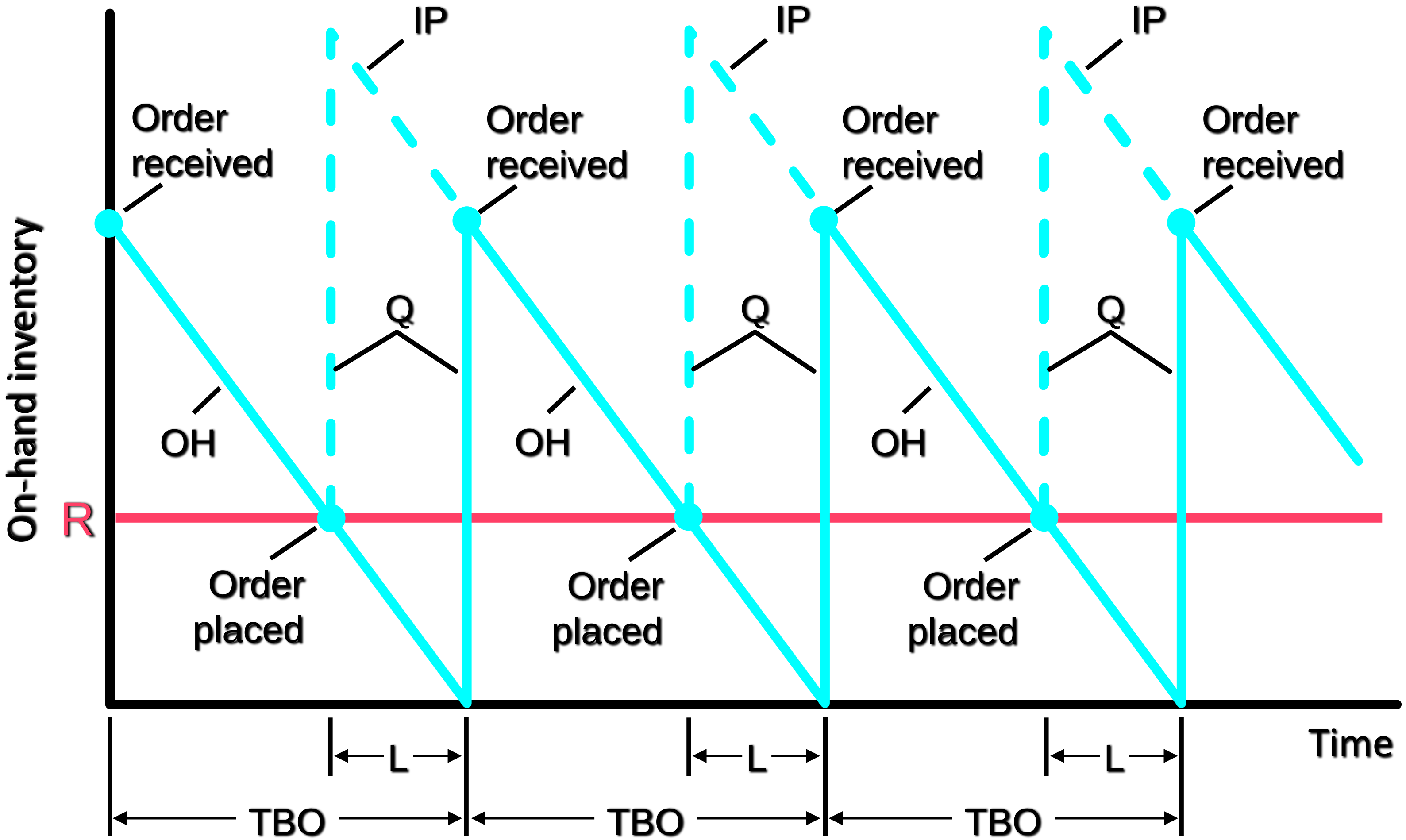


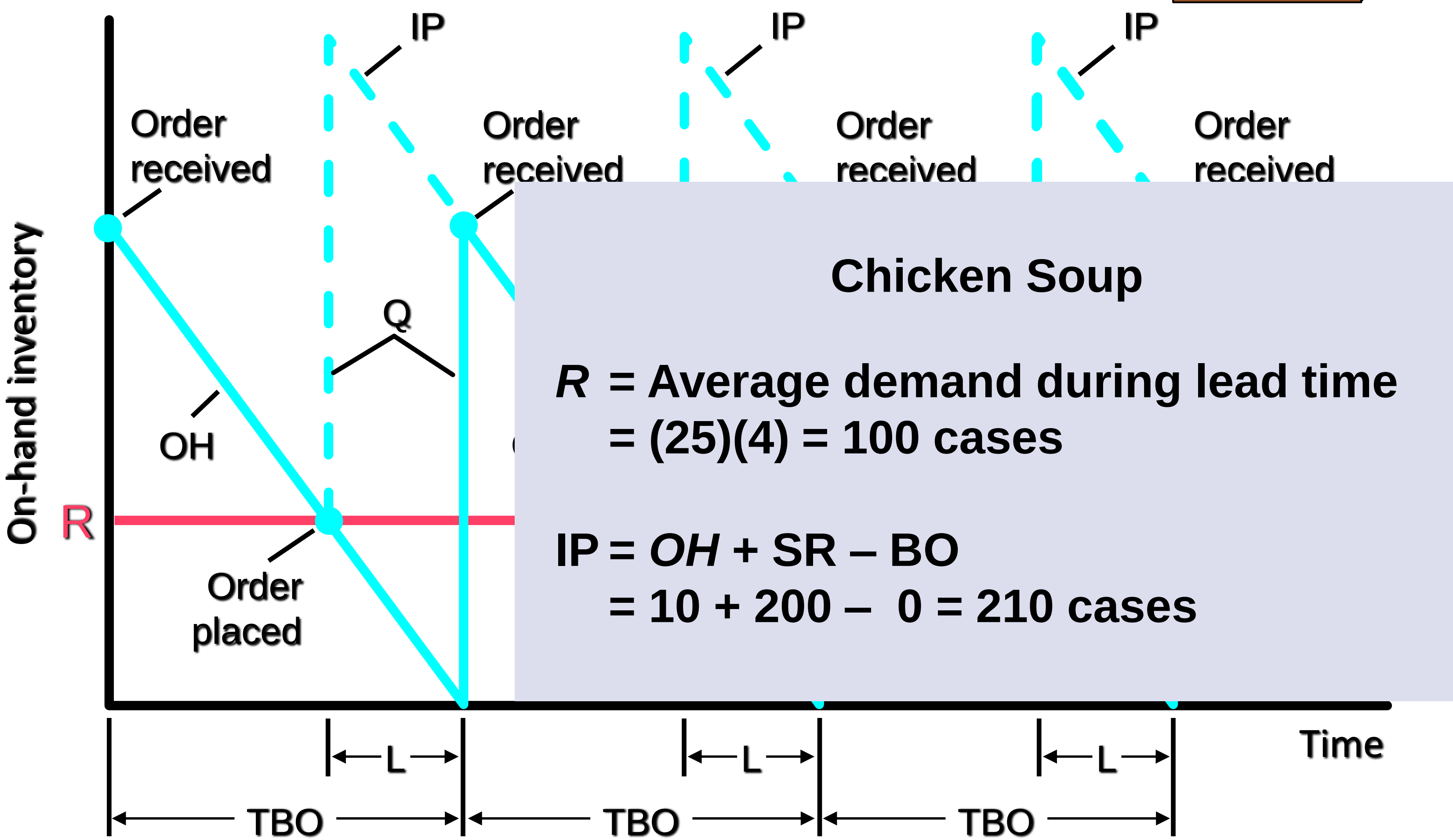
Figure 15.7

EXAMPLE

Determining Whether to Place an Order

Demand for chicken soup at a supermarket is always 25 cases a day and the lead time is always four days. The shelves were just restocked with chicken soup, leaving an on-hand inventory of only 10 cases. There are no backorders, but there is one open order for 200 cases. What is the inventory position? Should a new order be placed?

Continuous Review



Chicken Soup

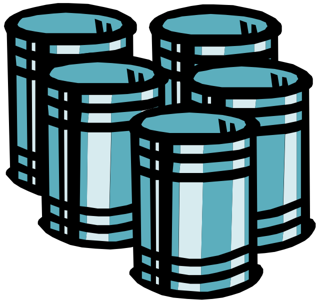
R **= Average demand during lead time**
= (25)(4) = 100 cases

IP = ***OH*** **+ SR – BO**
= 10 + 200 – 0 = 210 cases

Example 15.4

Special Inventory Models

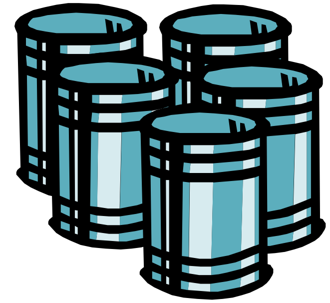
Quantity Discounts

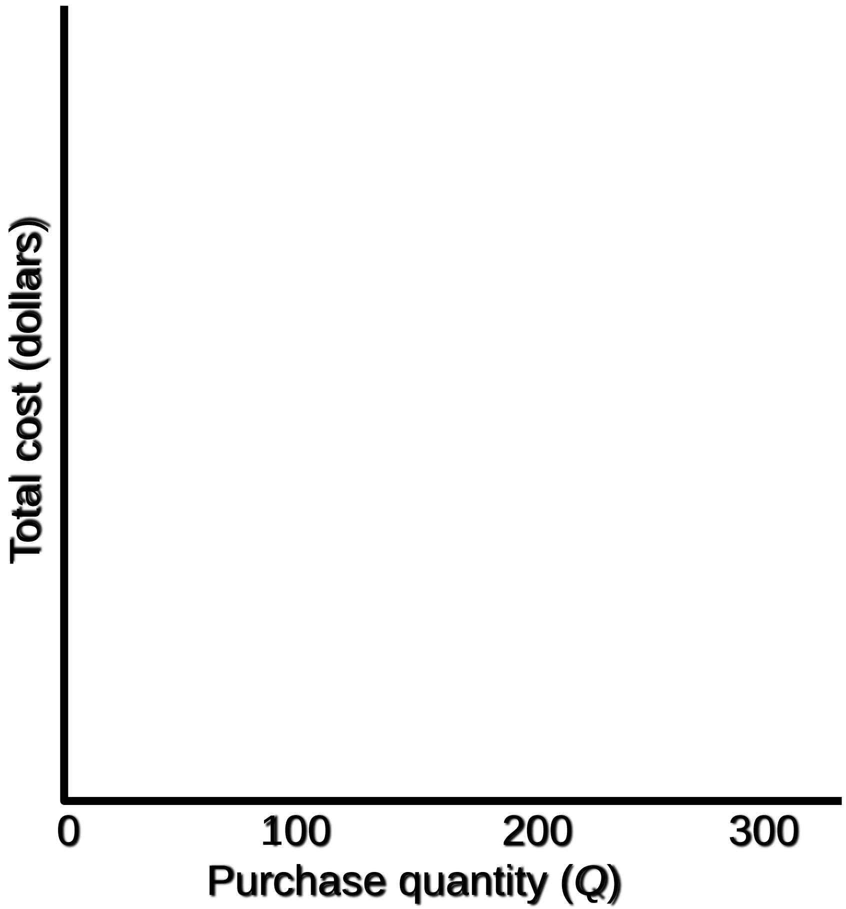


Special Inventory Models

Quantity Discounts

Figure E.3



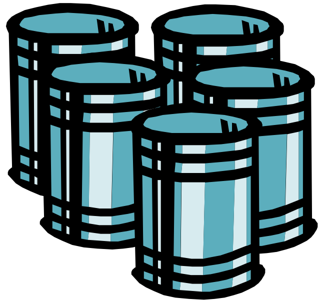


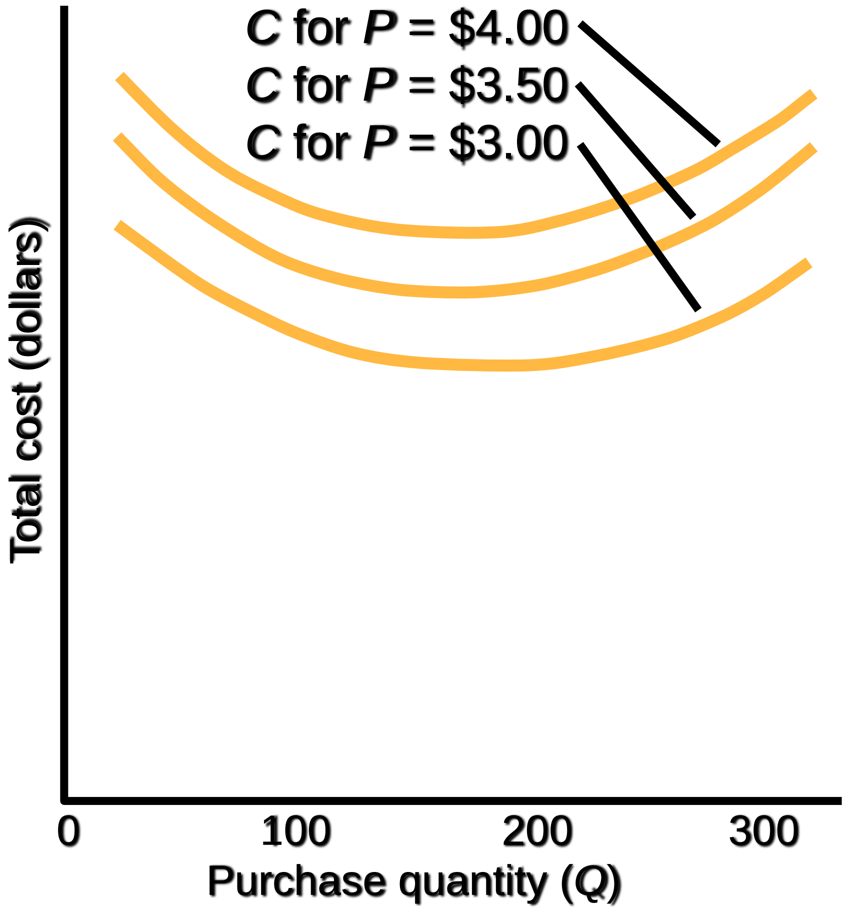
(a) Total cost curves with purchased materials added

Special Inventory Models

Quantity Discounts

Figure E.3



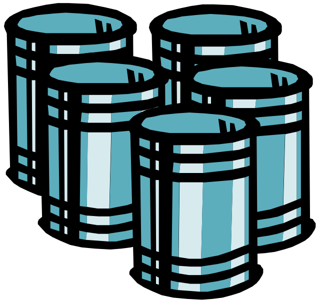


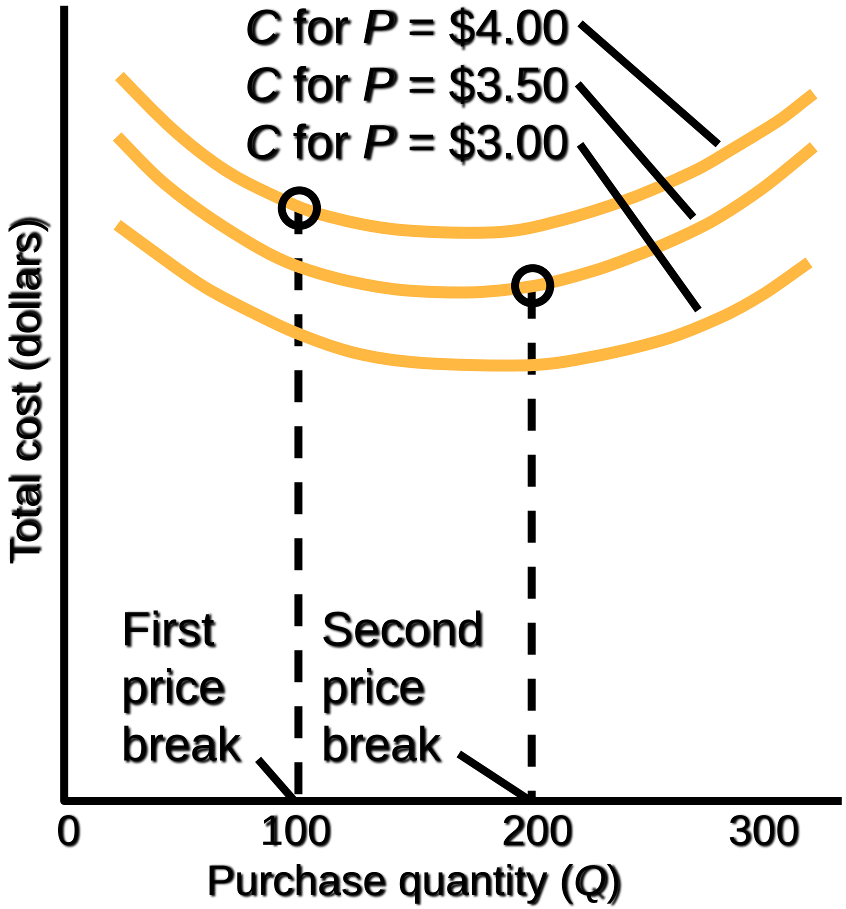
(a) Total cost curves with purchased materials added

Special Inventory Models

Quantity Discounts

Figure E.3



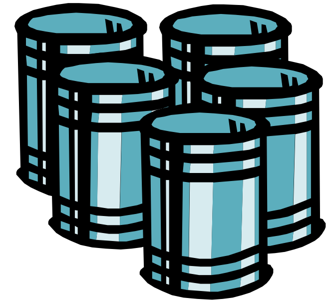


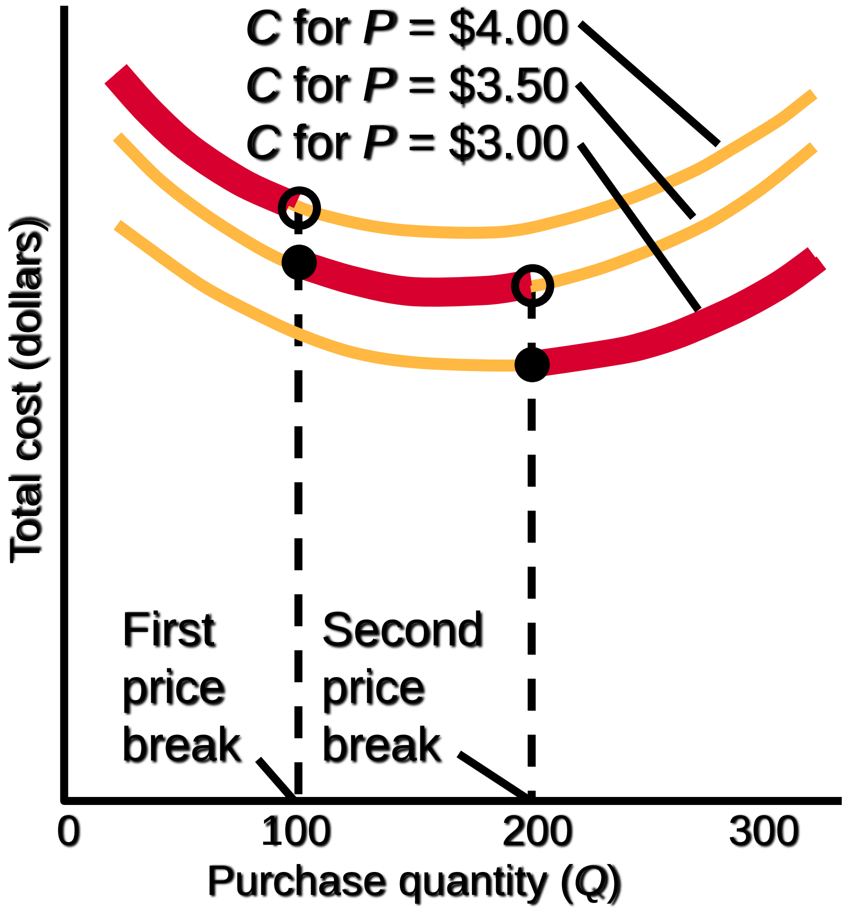
(a) Total cost curves with purchased materials added

Special Inventory Models

Quantity Discounts

Figure E.3



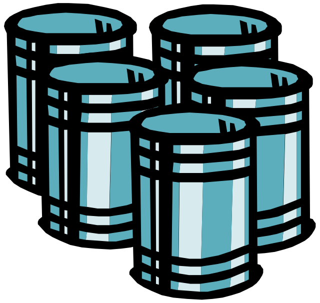


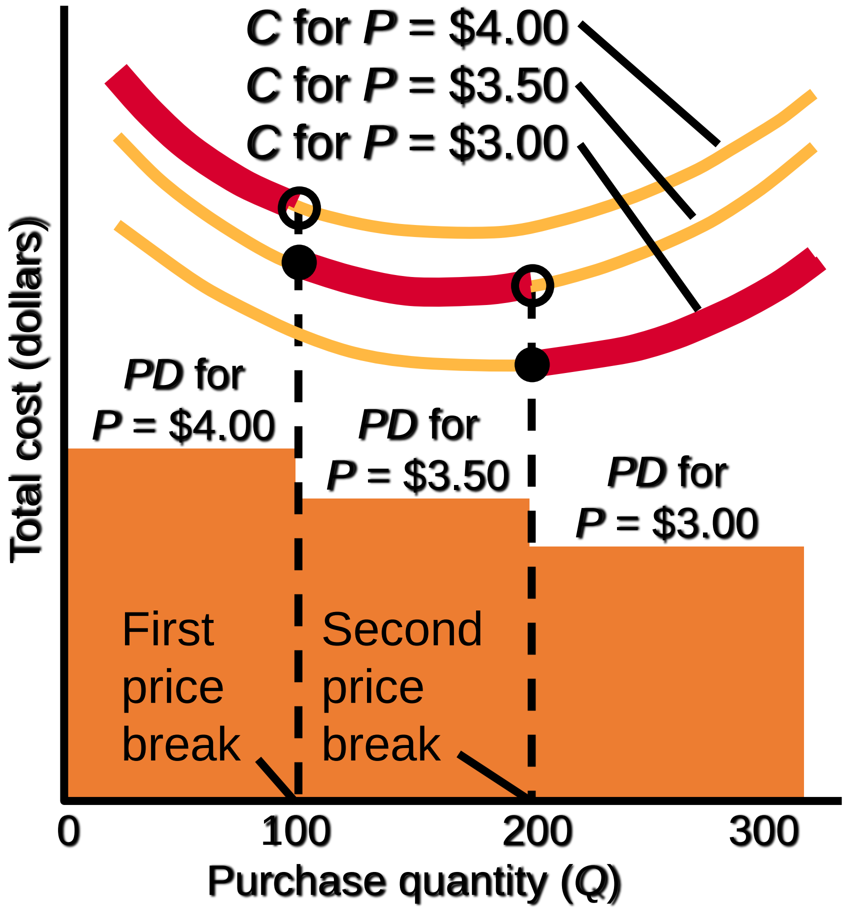
(a) Total cost curves with purchased materials added

Special Inventory Models

Quantity Discounts

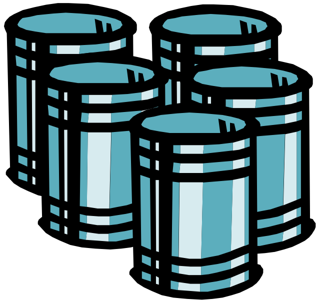
Figure E.3





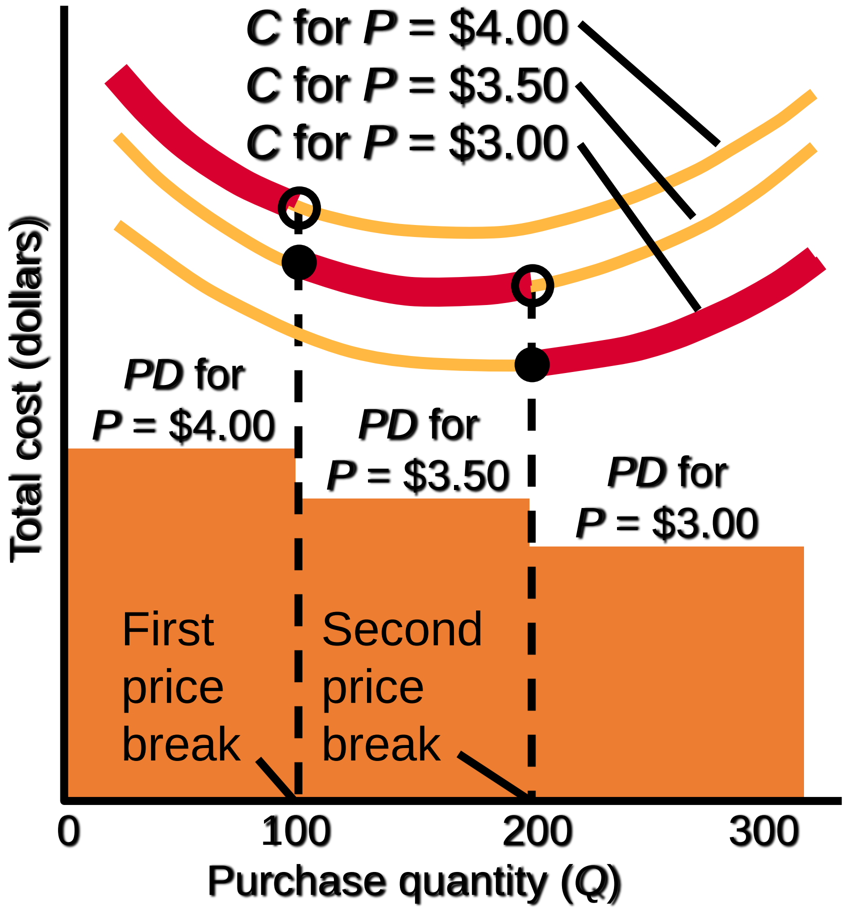
(a) Total cost curves with purchased materials added

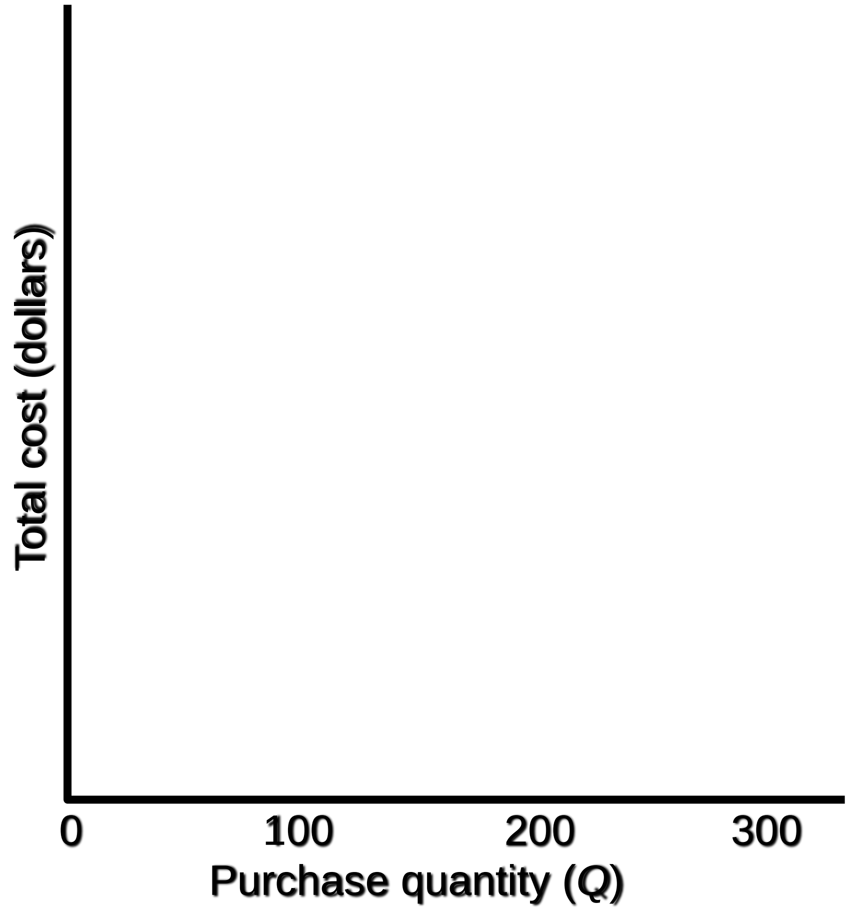
Special Inventory Models



Quantity Discounts

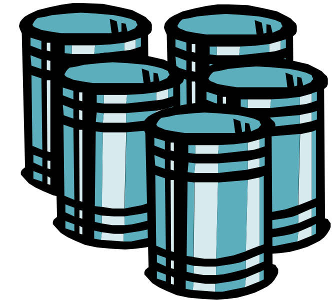
Figure E.3





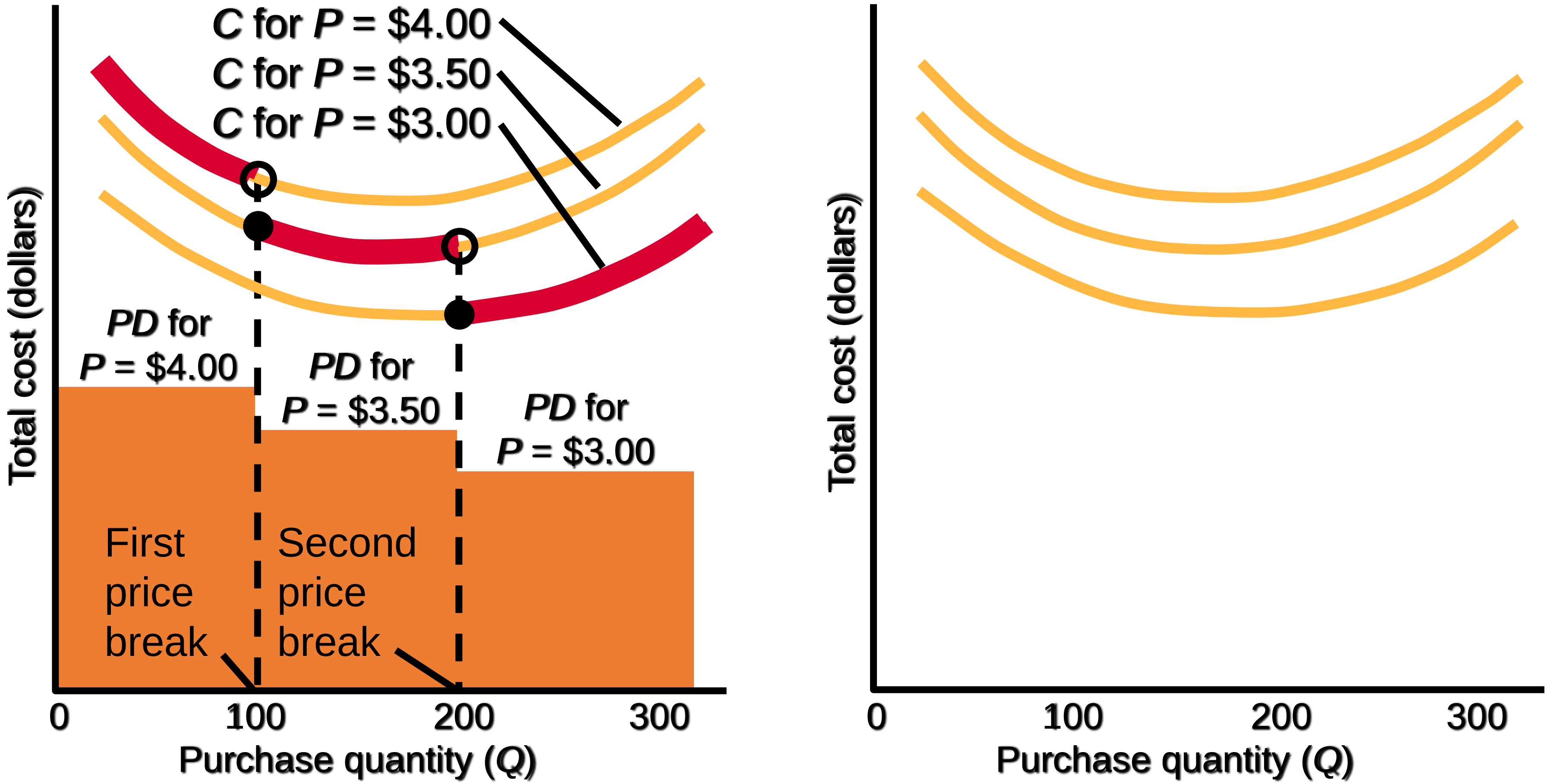
(b) EOQs and price break quantities

Special Inventory Models



Quantity Discounts

Figure E.3



(a) Total cost curves with purchased materials added

(b) EOQs and price break quantities

Special Inventory Models

Quantity Discounts

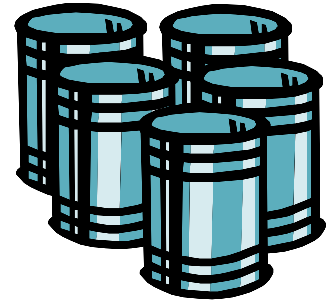
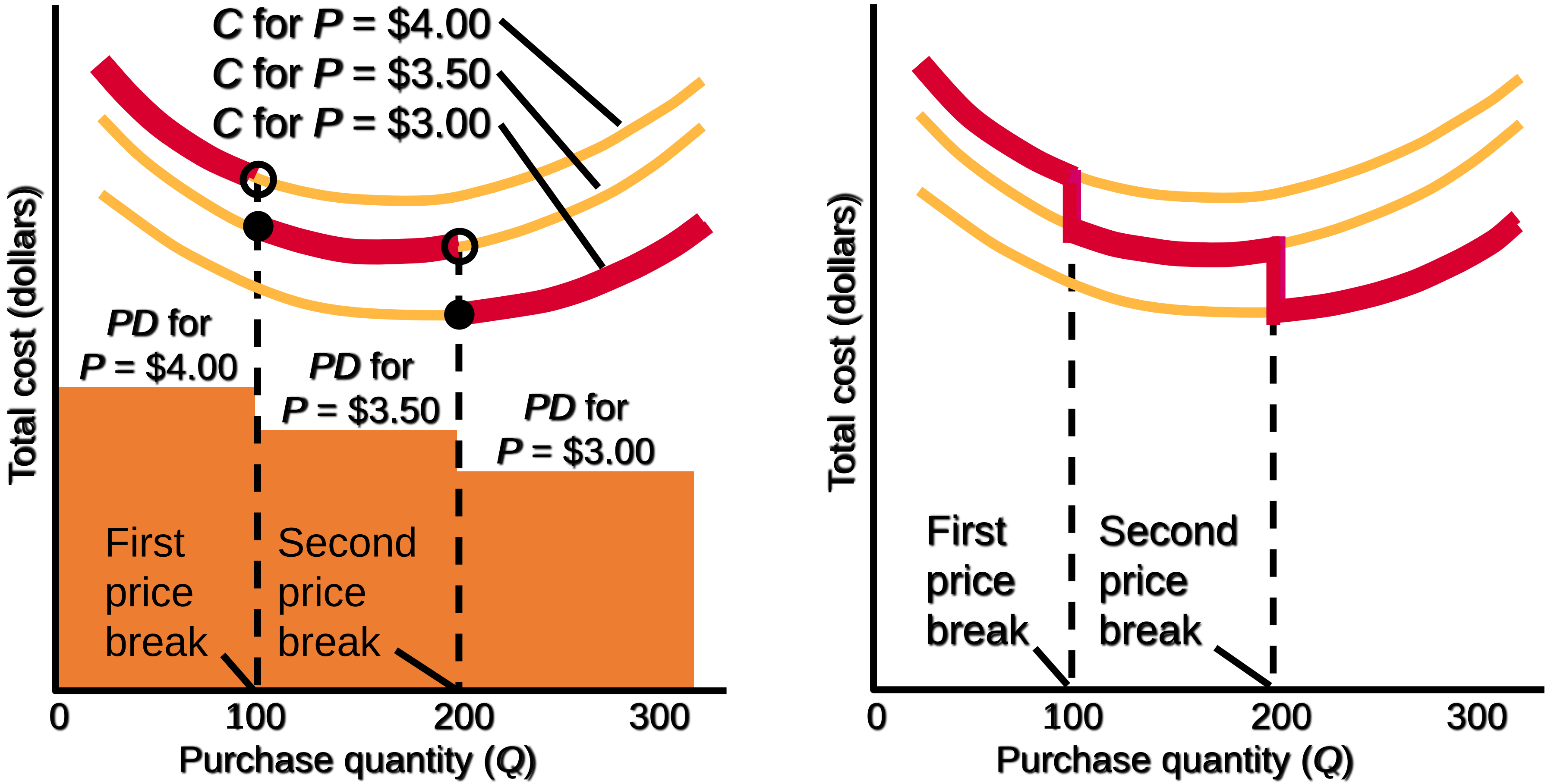
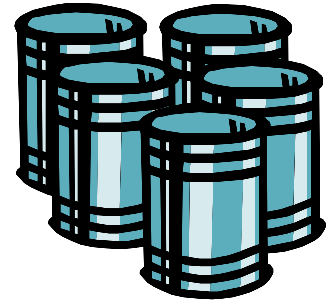


Figure E.3



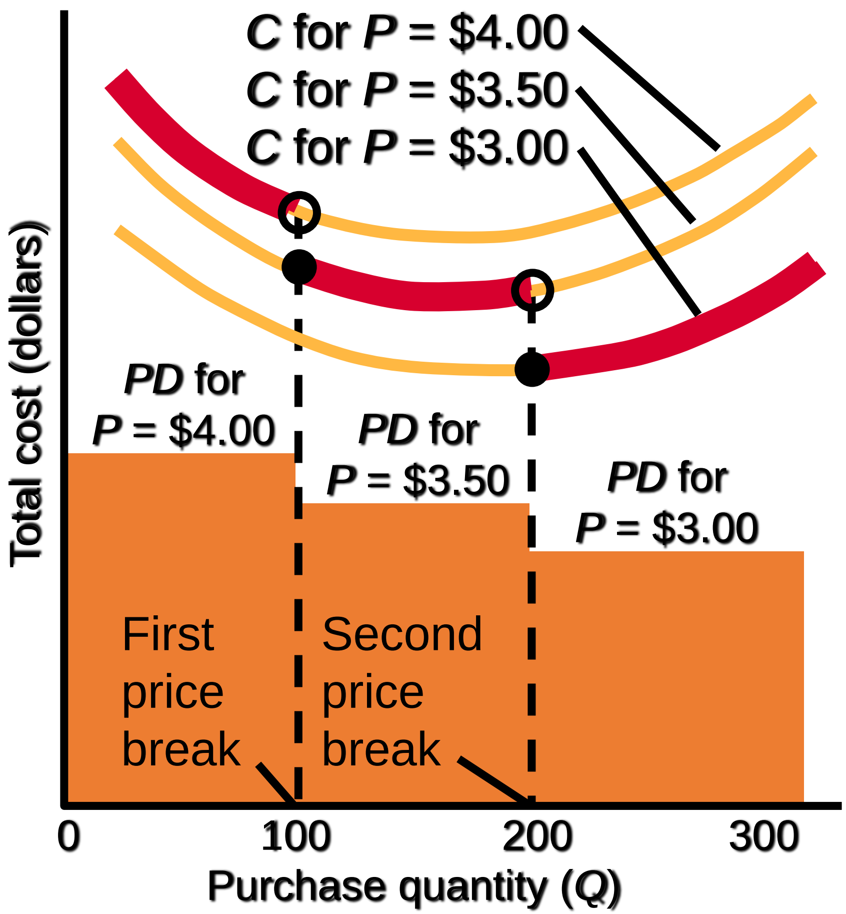
(b) EOQs and price break quantities

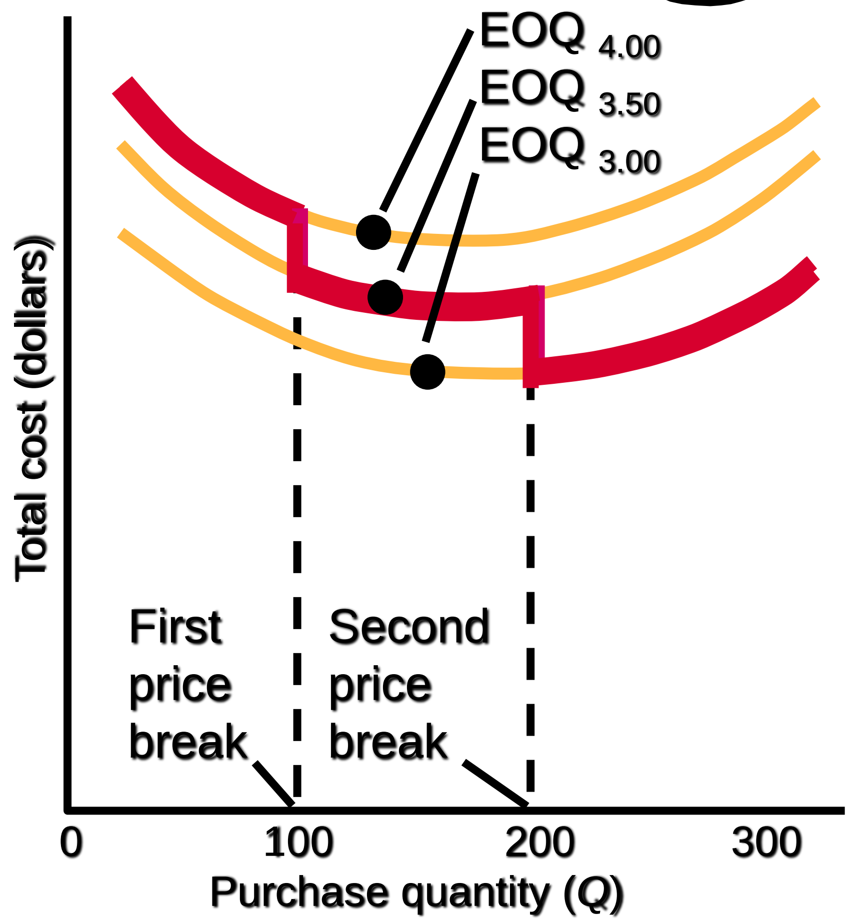
Special Inventory Models



Quantity Discounts

Figure E.3





(b) EOQs and price break quantities

Special Inventory Models

Quantity Discounts

Special Inventory Models

Quantity Discounts

Order Quantity	Price per Unit
0 – 299	$60.00
300 – 499	$58.80
500 or more	$57.00

Annual demand = 936 units
Ordering cost = $45
Holding cost = 25% of unit price

Example E.2

Special Inventory Models

Quantity Discounts

Order Quantity	Price per Unit
0 – 299	$60.00
300 – 499	$58.80
500 or more	$57.00

Annual demand = 936 units
Ordering cost = $45
Holding cost = 25% of unit price

$$EOQ_{57.00} = \sqrt{\frac{2DS}{H}}$$

Example E.2

Special Inventory Models

Quantity Discounts

Order Quantity	Price per Unit
0 – 299	$60.00
300 – 499	$58.80
500 or more	$57.00

Annual demand = 936 units
Ordering cost = $45
Holding cost = 25% of unit price

$$EOQ_{57.00} = \sqrt{\frac{2(936)(45)}{0.25(57.00)}}$$

Example E.2

Special Inventory Models

Quantity Discounts

Order Quantity	Price per Unit
0 – 299	$60.00
300 – 499	$58.80
500 or more	$57.00

Annual demand = 936 units
Ordering cost = $45
Holding cost = 25% of unit price

$$EOQ_{57.00} = \sqrt{\frac{2(936)(45)}{0.25(57.00)}}$$

Example E.2

Special Inventory Models

Quantity Discounts

Order Quantity	Price per Unit
0 – 299	$60.00
300 – 499	$58.80
500 or more	$57.00

Annual demand = 936 units
Ordering cost = $45
Holding cost = 25% of unit price

$EOQ_{57.00}$ = 77 units

Example E.2

Special Inventory Models

Quantity Discounts

Order Quantity	Price per Unit
0 – 299	$60.00
300 – 499	$58.80
500 or more	$57.00

Annual demand = 936 units
Ordering cost = $45
Holding cost = 25% of unit price

$EOQ_{57.00}$ = 77 units

Example E.2

Special Inventory Models

Quantity Discounts

Order Quantity	Price per Unit
0 – 299	$60.00
300 – 499	$58.80
500 or more	$57.00

Annual demand = 936 units
Ordering cost = $45
Holding cost = 25% of unit price

~~$EOQ_{57.00}$ = 77 units~~

Example E.2

Special Inventory Models

Quantity Discounts

Order Quantity	Price per Unit
0 – 299	$60.00
300 – 499	$58.80
500 or more	$57.00

Annual demand = 936 units
Ordering cost = $45
Holding cost = 25% of unit price

~~$EOQ_{57.00}$ = 77 units~~ $EOQ_{58.80}$ = 76 units

Example E.2

Special Inventory Models

Quantity Discounts

Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units
Ordering cost = \$45
Holding cost = 25% of unit price

~~$EOQ_{57.00}$ = 77 units~~ $EOQ_{58.80}$ = 76 units

Example E.2

Special Inventory Models

Quantity Discounts

Order Quantity	Price per Unit
0 – 299	$60.00
300 – 499	$58.80
500 or more	$57.00

Annual demand = 936 units
Ordering cost = $45
Holding cost = 25% of unit price

$EOQ_{57.00}$ = 77 units

$EOQ_{58.80}$ = 76 units

Example E.2

Special Inventory Models

Quantity Discounts

Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units
Ordering cost = \$45
Holding cost = 25% of unit price

~~$EOQ_{57.00}$ = 77 units~~ ~~$EOQ_{58.80}$ = 76 units~~ $EOQ_{60.00}$ = 75 units

Example E.2

Special Inventory Models

Quantity Discounts

Order Quantity	Price per Unit
0 – 299	$60.00
300 – 499	$58.80
500 or more	$57.00

Annual demand = 936 units
Ordering cost = $45
Holding cost = 25% of unit price

$EOQ_{57.00}$ = 77 units $EOQ_{58.80}$ = 76 units $EOQ_{60.00}$ = 75 units

Example E.2

Special Inventory Models

Quantity Discounts

Order Quantity	Price per Unit
0 – 299	$60.00
300 – 499	$58.80
500 or more	$57.00

Annual demand = 936 units
Ordering cost = $45
Holding cost = 25% of unit price

~~$EOQ_{57.00}$ = 77 units~~ ~~$EOQ_{58.80}$ = 76 units~~ $EOQ_{60.00}$ = 75 units

Example E.2

Special Inventory Models

Quantity Discounts

Order Quantity	Price per Unit
0 – 299	$60.00
300 – 499	$58.80
500 or more	$57.00

Annual demand = 936 units
Ordering cost = $45
Holding cost = 25% of unit price

~~$EOQ_{57.00}$ = 77 units~~ ~~$EOQ_{58.80}$ = 76 units~~ $EOQ_{60.00}$ = 75 units

$$C = \frac{Q}{2}(H) + \frac{D}{Q}(S) + PD$$

Example E.2

Special Inventory Models

Quantity Discounts

Order Quantity	Price per Unit
0 – 299	$60.00
300 – 499	$58.80
500 or more	$57.00

Annual demand = 936 units
Ordering cost = $45
Holding cost = 25% of unit price

~~$EOQ_{57.00}$ = 77 units~~ ~~$EOQ_{58.80}$ = 76 units~~ $EOQ_{60.00}$ = 75 units

$$C_{75} = \frac{75}{2}[(0.25)(\$60.00)] + \frac{936}{75}(\$45) + \$60.00(936)$$

Example E.2

Special Inventory Models

Quantity Discounts

Order Quantity	Price per Unit
0 – 299	$60.00
300 – 499	$58.80
500 or more	$57.00

Annual demand = 936 units
Ordering cost = $45
Holding cost = 25% of unit price

~~$EOQ_{57.00}$ = 77 units~~ ~~$EOQ_{58.80}$ = 76 units~~ $EOQ_{60.00}$ = 75 units

C_{75} = $57,284

Example E.2

Special Inventory Models

Quantity Discounts

Order Quantity	Price per Unit
0 – 299	$60.00
300 – 499	$58.80
500 or more	$57.00

Annual demand = 936 units
Ordering cost = $45
Holding cost = 25% of unit price

$EOQ_{57.00}$ = 77 units $EOQ_{58.80}$ = 76 units $EOQ_{60.00}$ = 75 units

C_{75} = $57,284

$$C_{300} = \frac{300}{2}[(0.25)(\$58.80)] + \frac{936}{300}(\$45) + \$58.80(936)$$

Example E.2

Special Inventory Models

Quantity Discounts

Order Quantity	Price per Unit
0 – 299	\$60.00
300 – 499	\$58.80
500 or more	\$57.00

Annual demand = 936 units
Ordering cost = \$45
Holding cost = 25% of unit price

~~$EOQ_{57.00} = 77$ units~~ ~~$EOQ_{58.80} = 76$ units~~ $EOQ_{60.00} = 75$ units

$C_{75} = \$57,284$

$C_{300} = \$57,382$

Example E.2

Special Inventory Models

Quantity Discounts

Order Quantity	Price per Unit
0 – 299	$60.00
300 – 499	$58.80
500 or more	$57.00

Annual demand = 936 units
Ordering cost = $45
Holding cost = 25% of unit price

~~$EOQ_{57.00}$ = 77 units~~ ~~$EOQ_{58.80}$ = 76 units~~ $EOQ_{60.00}$ = 75 units

C_{75} = $57,284

C_{300} = $57,382

$$C_{500} = \frac{500}{2}[(0.25)(\$57.00)] + \frac{936}{500}(\$45) + \$57.00(936)$$

Example E.2

Special Inventory Models

Quantity Discounts

Order Quantity	Price per Unit
0 – 299	$60.00
300 – 499	$58.80
500 or more	$57.00

Annual demand = 936 units
Ordering cost = $45
Holding cost = 25% of unit price

$EOQ_{57.00} = 77$ units $EOQ_{58.80} = 76$ units $EOQ_{60.00} = 75$ units

$C_{75} = \$57,284$

$C_{300} = \$57,382$

$C_{300} = \$56,999$

Example E.2

Special Inventory Models

Quantity Discounts

Order Quantity	Price per Unit
0 – 299	$60.00
300 – 499	$58.80
500 or more	$57.00

Annual demand = 936 units
Ordering cost = $45
Holding cost = 25% of unit price

~~$EOQ_{57.00}$ = 77 units~~ ~~$EOQ_{58.80}$ = 76 units~~ $EOQ_{60.00}$ = 75 units

C_{75} = $57,284

C_{300} = $57,382

C_{500} = $56,999

Example E.2

Discount

Whole Nature Foods sells a gluten-free product for which the annual demand is 5,000 boxes. At the moment, it is paying \$6.40 for each box; carrying cost is 25% of the unit cost; ordering costs are \$25. A new supplier has offered to sell the same item for \$6.00 if Whole Nature Foods buys at least 3,000 boxes per order. Should the firm stick with the old supplier, or take advantage of the new quantity discount?

Special Inventory Models

One-Period Decisions

Special Inventory Models

One-Period Decisions

Demand	10	20	30	40	50
Demand Probability	0.2	0.3	0.3	0.1	0.1

Profit per ornament during season = $10
Loss per ornament after season = $5

Example E.3

Special Inventory Models

One-Period Decisions

Demand	10	20	30	40	50
Demand Probability	0.2	0.3	0.3	0.1	0.1

Profit per ornament during season = $10
Loss per ornament after season = $5

			D		
Q	10	20	30	40	50
10					
20					
30					
40					
50					

For $Q = D$
Payoff = pQ

Example E.3

Special Inventory Models

One-Period Decisions

Demand	10	20	30	40	50
Demand Probability	0.2	0.3	0.3	0.1	0.1

Profit per ornament during season = $10
Loss per ornament after season = $5

	D				
Q	10	20	30	40	50
10					
20					
30					
40					
50					

For *Q* = *D*
Payoff = ($10)(10)

Example E.3

Special Inventory Models

One-Period Decisions

Demand	10	20	30	40	50
Demand Probability	0.2	0.3	0.3	0.1	0.1

Profit per ornament during season = $10
Loss per ornament after season = $5

	D				
Q	10	20	30	40	50
10	$100				
20					
30					
40					
50					

For $Q = D$
Payoff = $100

Example E.3

Special Inventory Models

One-Period Decisions

Demand	10	20	30	40	50
Demand Probability	0.2	0.3	0.3	0.1	0.1

Profit per ornament during season = $10
Loss per ornament after season = $5

	D				
Q	10	20	30	40	50
10	$100	$100	$100	$100	$100
20					
30					
40					
50					

For $Q \leq D$
Payoff = pQ

Example E.3

Special Inventory Models

One-Period Decisions

Demand	10	20	30	40	50
Demand Probability	0.2	0.3	0.3	0.1	0.1

Profit per ornament during season = $10
Loss per ornament after season = $5

	D				
Q	10	20	30	40	50
10	$100	$100	$100	$100	$100
20		200	200	200	200
30			300	300	300
40				400	400
50					500

For $Q \leq D$
Payoff = pQ

Example E.3

Special Inventory Models

One-Period Decisions

Demand	10	20	30	40	50
Demand Probability	0.2	0.3	0.3	0.1	0.1

Profit per ornament during season = $10
Loss per ornament after season = $5

	D				
Q	**10**	**20**	**30**	**40**	**50**
10	$100	$100	$100	$100	$100
20		200	200	200	200
30			300	300	300
40				400	400
50					500

For $Q > D$
Payoff = $pD - l(Q - D)$

Example E.3

Special Inventory Models

One-Period Decisions

Demand	10	20	30	40	50
Demand Probability	0.2	0.3	0.3	0.1	0.1

Profit per ornament during season = $10
Loss per ornament after season = $5

	D				
Q	10	20	30	40	50
10	$100	$100	$100	$100	$100
20		200	200	200	200
30			300	300	300
40				400	400
50					500

For *Q* > *D*
Payoff = ($10)(30)
– ($5)(40 – 30)

Example E.3

Special Inventory Models

One-Period Decisions

Demand	10	20	30	40	50
Demand Probability	0.2	0.3	0.3	0.1	0.1

Profit per ornament during season = $10
Loss per ornament after season = $5

	D				
Q	10	20	30	40	50
10	$100	$100	$100	$100	$100
20		200	200	200	200
30			300	300	300
40				400	400
50					500

For *Q* > *D*
Payoff = ($10)(30)
– ($5)(40 – 30)

Example E.3

Special Inventory Models

One-Period Decisions

Demand	10	20	30	40	50
Demand Probability	0.2	0.3	0.3	0.1	0.1

Profit per ornament during season = $10
Loss per ornament after season = $5

	D				
Q	10	20	30	40	50
10	$100	$100	$100	$100	$100
20		200	200	200	200
30			300	300	300
40				400	400
50					500

For *Q* > *D*
Payoff = $250

Example E.3

Special Inventory Models

One-Period Decisions

Demand	10	20	30	40	50
Demand Probability	0.2	0.3	0.3	0.1	0.1

Profit per ornament during season = $10
Loss per ornament after season = $5

	D				
Q	10	20	30	40	50
10	$100	$100	$100	$100	$100
20		200	200	200	200
30			300	300	300
40			250	400	400
50					500

For *Q* > *D*
Payoff = $250

Example E.3

Special Inventory Models

One-Period Decisions

Demand	10	20	30	40	50
Demand Probability	0.2	0.3	0.3	0.1	0.1

Profit per ornament during season = $10
Loss per ornament after season = $5

Q	*D* 10	20	30	40	50
10	$100	$100	$100	$100	$100
20	50	200	200	200	200
30	0	150	300	300	300
40	–50	100	250	400	400
50	–100	50	200	350	500

For $Q > D$
Payoff = $pD - l(Q - D)$

Example E.3

Special Inventory Models

One-Period Decisions

Demand	10	20	30	40	50
Demand Probability	0.2	0.3	0.3	0.1	0.1

F
L

Expected payoff_{30} =

			D		
Q	10	20	30	40	50
10	$100	$100	$100	$100	$100
20	50	200	200	200	200
30	0	150	300	300	300
40	–50	100	250	400	400
50	–100	50	200	350	500

Example E.3

Special Inventory Models

One-Period Decisions

Demand	10	20	30	40	50
Demand Probability	0.2	0.3	0.3	0.1	0.1

F
L

Expected payoff_{30} = 0.2($0)

	D				
Q	10	20	30	40	50
10	$100	$100	$100	$100	$100
20	50	200	200	200	200
30	0	150	300	300	300
40	–50	100	250	400	400
50	–100	50	200	350	500

Example E.3

Special Inventory Models

One-Period Decisions

Demand	10	20	30	40	50
Demand Probability	0.2	0.3	0.3	0.1	0.1

P

L

Expected payoff$_{30}$ = 0.2($0) + 0.3($150)

	D				
Q	10	20	30	40	50
10	$100	$100	$100	$100	$100
20	50	200	200	200	200
30	0	150	300	300	300
40	–50	100	250	400	400
50	–100	50	200	350	500

Example E.3

Special Inventory Models

One-Period Decisions

Demand	10	20	30	40	50
Demand Probability	0.2	0.3	0.3	0.1	0.1

Expected payoff_{30} = 0.2($0) + 0.3($150) + 0.3($300)

	D				
Q	10	20	30	40	50
10	$100	$100	$100	$100	$100
20	50	200	200	200	200
30	0	150	300	300	300
40	–50	100	250	400	400
50	–100	50	200	350	500

Example E.3

Special Inventory Models

One-Period Decisions

Demand	10	20	30	40	50
Demand Probability	0.2	0.3	0.3	0.1	0.1

Expected payoff_{30} = 0.2($0) + 0.3($150) + 0.3($300) + 0.1($300)

	D				
Q	10	20	30	40	50
10	$100	$100	$100	$100	$100
20	50	200	200	200	200
30	0	150	300	300	300
40	–50	100	250	400	400
50	–100	50	200	350	500

Example E.3

Special Inventory Models

One-Period Decisions

Demand	10	20	30	40	50
Demand Probability	0.2	0.3	0.3	0.1	0.1

Expected payoff_{30} = 0.2($0) + 0.3($150) + 0.3($300) + 0.1($300) + 0.1($300)

	D				
Q	**10**	**20**	**30**	**40**	**50**
10	**$100**	**$100**	**$100**	**$100**	**$100**
20	**50**	**200**	**200**	**200**	**200**
30	**0**	**150**	**300**	**300**	**300**
40	**–50**	**100**	**250**	**400**	**400**
50	**–100**	**50**	**200**	**350**	**500**

Example E.3

Special Inventory Models

One-Period Decisions

Demand	10	20	30	40	50
Demand Probability	0.2	0.3	0.3	0.1	0.1

Expected payoff_{30} = $195

	D				
Q	10	20	30	40	50
10	$100	$100	$100	$100	$100
20	50	200	200	200	200
30	0	150	300	300	300
40	–50	100	250	400	400
50	–100	50	200	350	500

Example E.3

Special Inventory Models

One-Period Decisions

Demand	10	20	30	40	50
Demand Probability	0.2	0.3	0.3	0.1	0.1

P
L

Expected payoff$_{30}$ = $195

Q	*D* 10	20	30	40	50	Expected Payoff
10	$100	$100	$100	$100	$100	
20	50	200	200	200	200	
30	0	150	300	300	300	195
40	–50	100	250	400	400	
50	–100	50	200	350	500	

Example E.3

Uncertain Demand

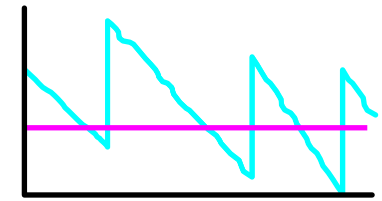
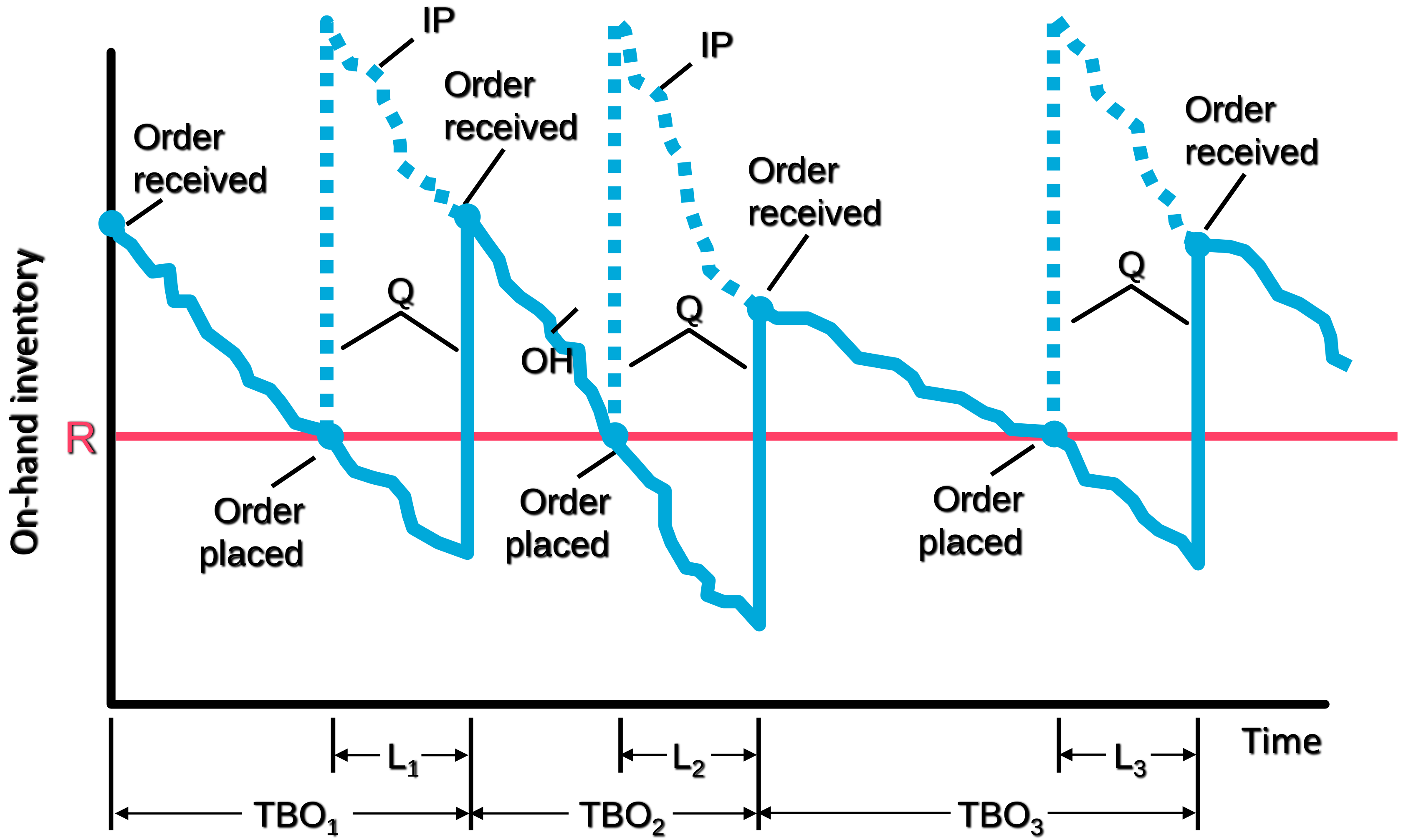


Figure 15.8



Reorder Point / Safety Stock

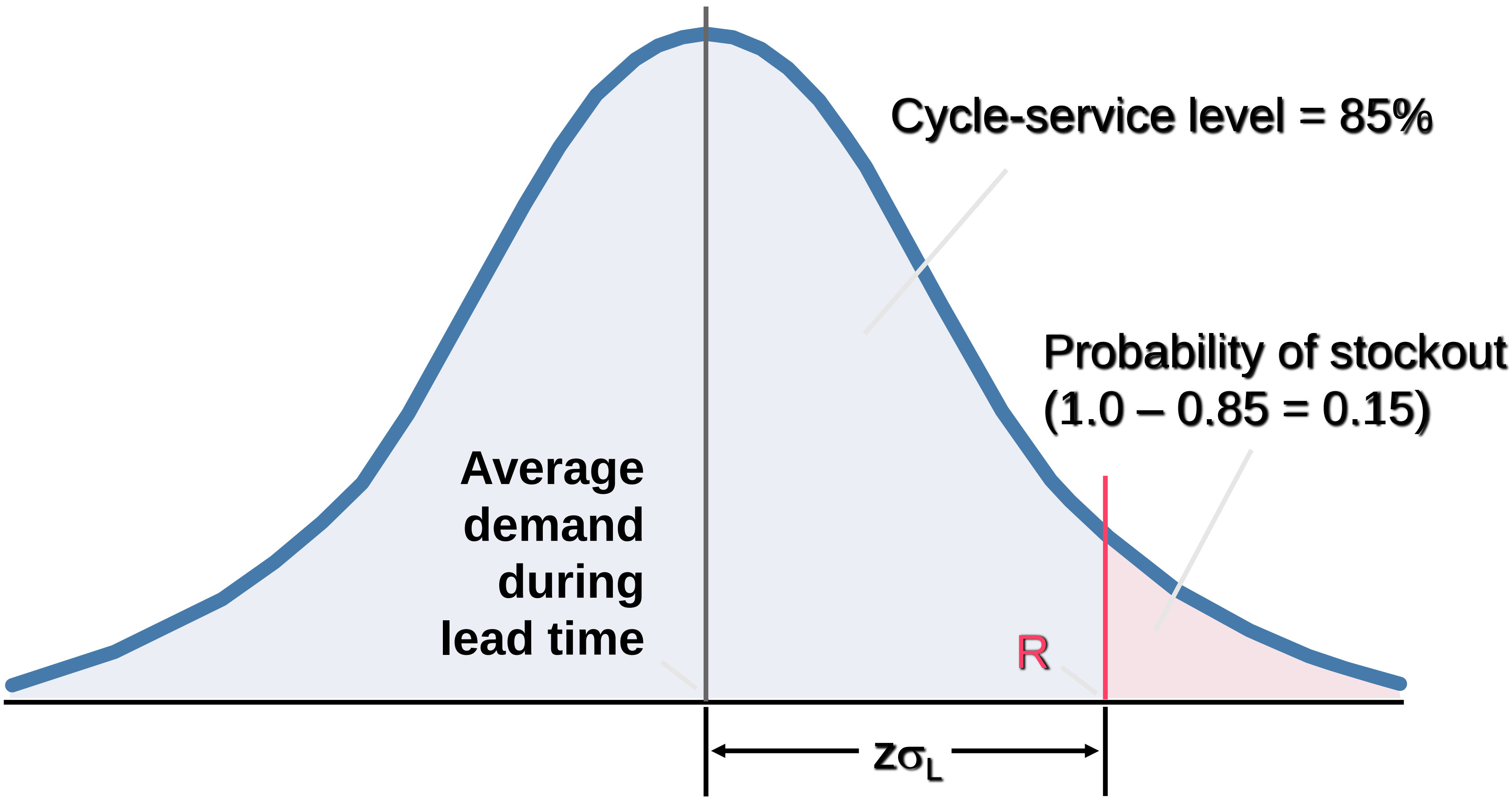


Figure 15.9

EXAMPLE

Records show that the demand for dishwasher detergent during the lead time is normally distributed, with an average of 250 boxes and *variance l = 22*.

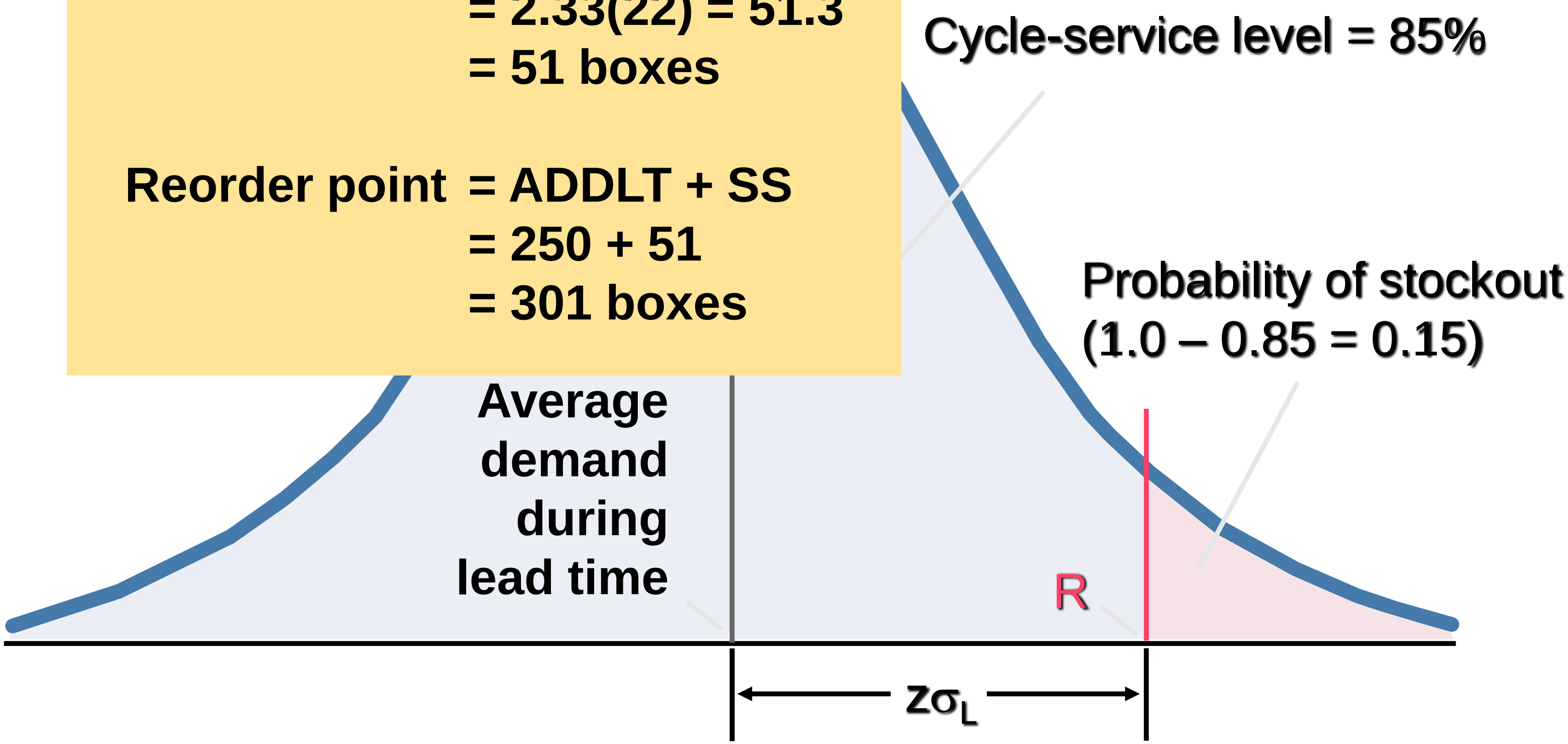
What safety stock should be carried for a 99 percent cycle-service level? What is R?

Reorder Point / Safety Stock

Safety Stock/R

Safety stock $= z\sigma_L$
$= 2.33(22) = 51.3$
$= 51$ boxes

Reorder point = ADDLT + SS
= 250 + 51
= 301 boxes



Example 15.5

Lead Time Distributions

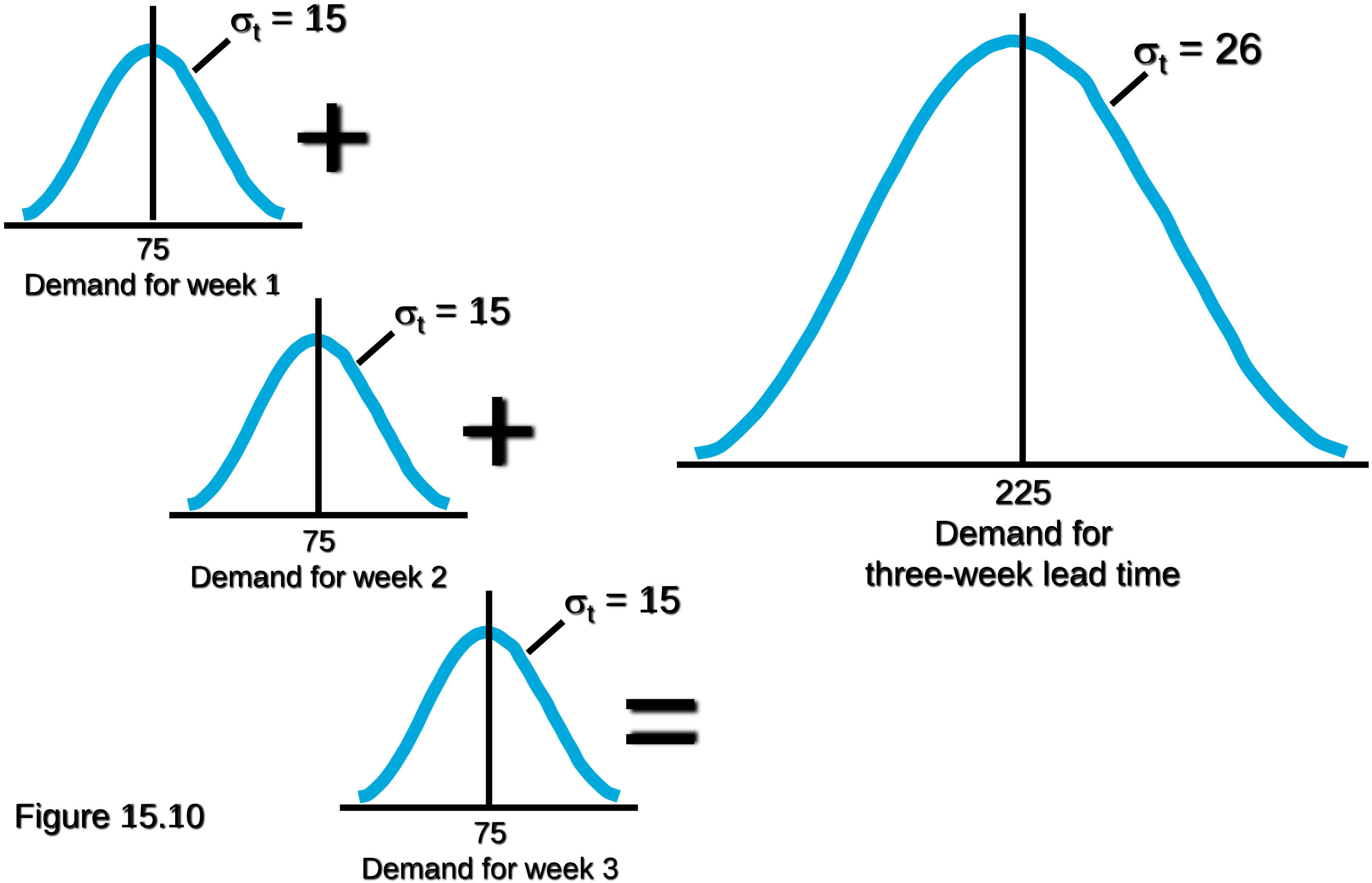
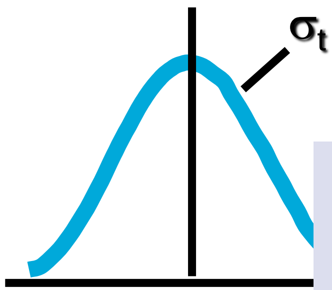


Figure 15.10

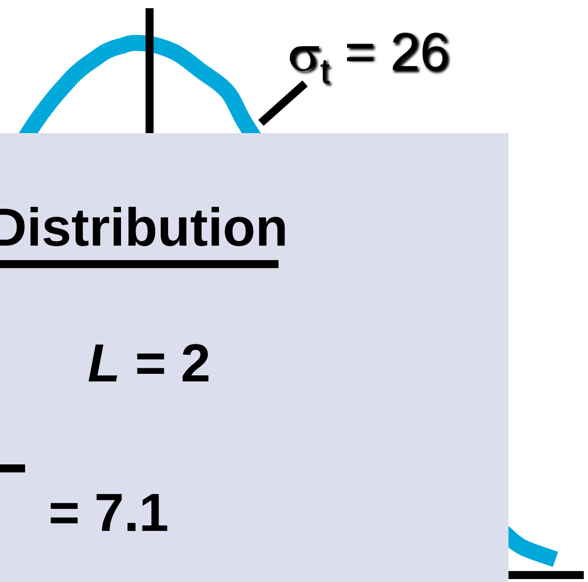
Example Finding the safety stock and R When the Demand Distribution for Lead Time must Be Developped

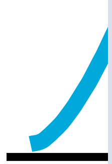
- Let us return to the bird feeder example. Suppose that the average demand is 18 units per week \
- with a standard deviation of 5 units. The lead time is constant at two weeks. Determine the safety stock and reorder point if management wants a 90 percent cycle-service level. What is the j total cost of the Q system?

Lead Time Distributions



75
Demand for week





Dem

<u>Bird feeder Lead Time Distribution</u>

t = 1 week d = 18 L = 2

$$\sigma_L = \sigma_t \sqrt{L} = 5\sqrt{2} = 7.1$$

Safety stock $= z\sigma_L = 1.28(7.1) = 9.1$ **or 9 units**

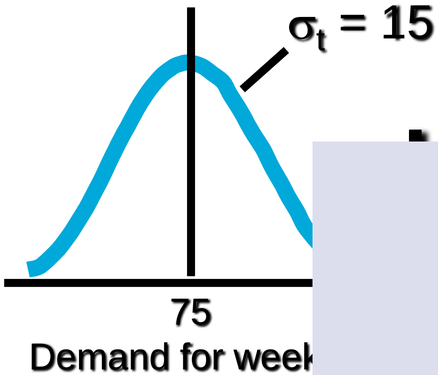
Reorder point $= dL +$ **Safety stock**
$= 2(18) + 9 = 45$ **units**

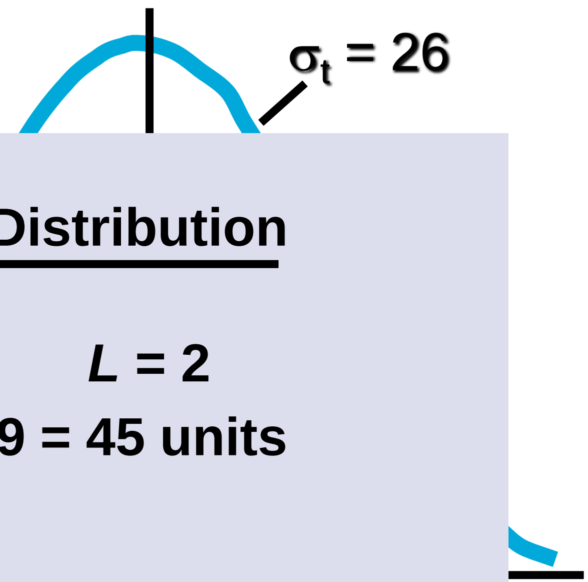
Example 15.6

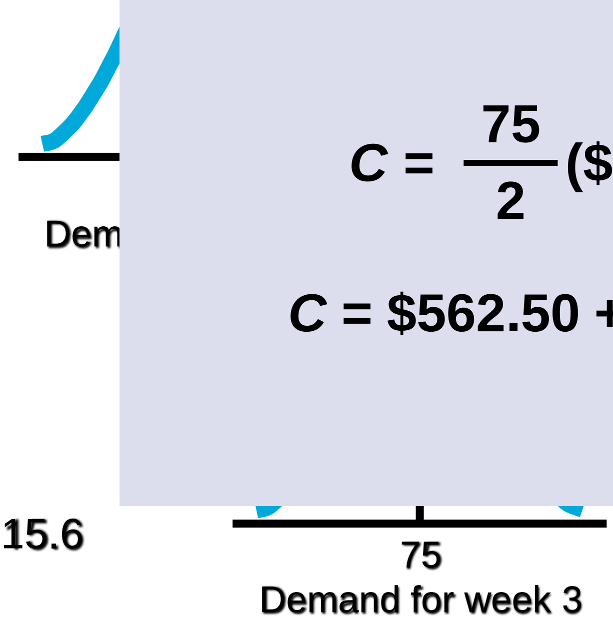
75
Demand for week 3

	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Lead Time Distributions







Bird feeder Lead Time Distribution

t = 1 week d = 18 L = 2

Reorder point = 2(18) + 9 = 45 units

$$C = \frac{75}{2}(\$15) + \frac{936}{75}(\$45) + 9(\$15)$$

$$C = \$562.50 + \$561.60 + \$135 = \$1259.10$$

Example 15.6

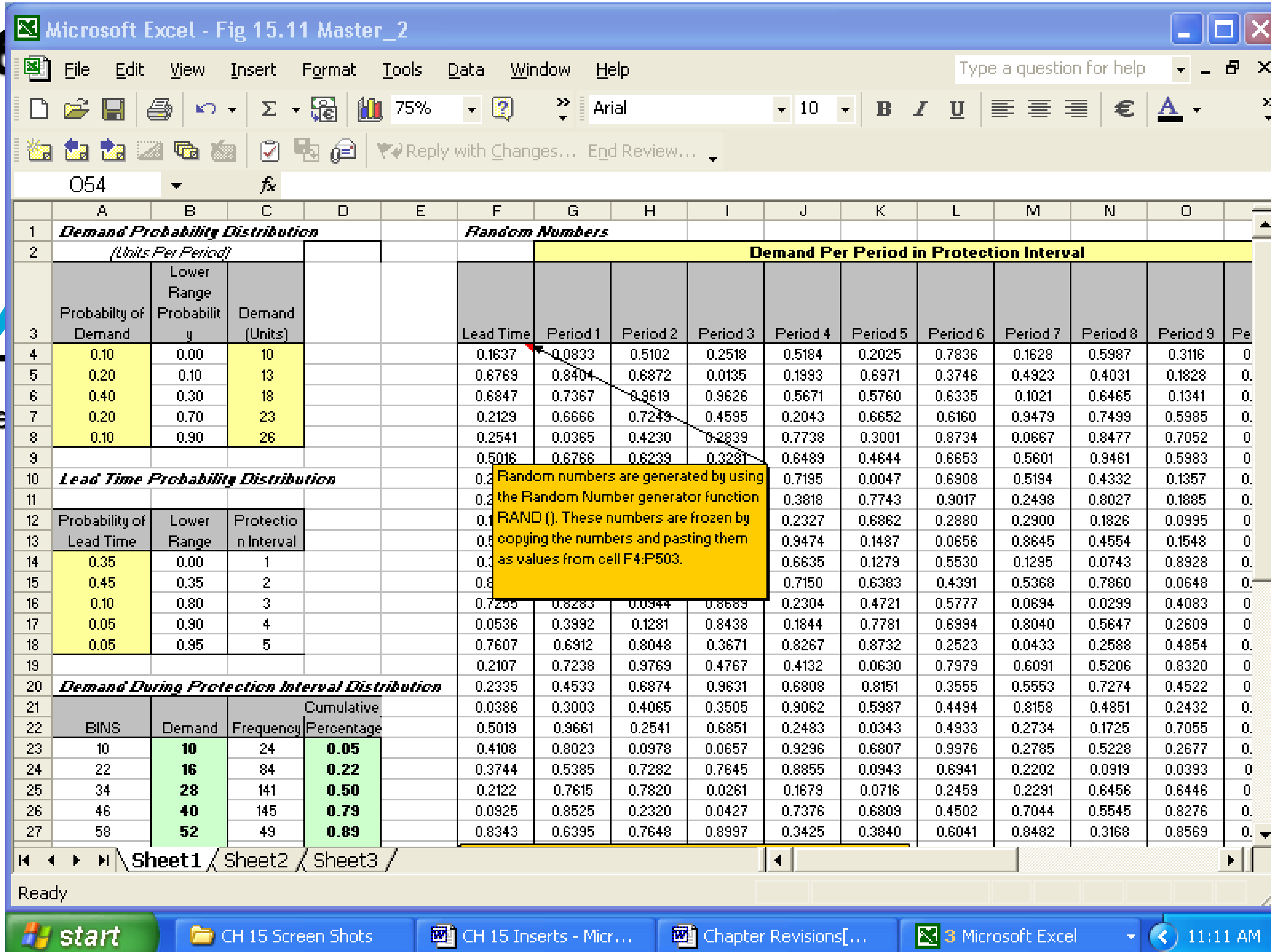
Lead Time Distributions

TABLE 15.1 PROBABILITY DISTRIBUTION FOR LEAD TIME

Lead Time (weeks)	Probability for Lead Time
1	0.35
2	0.45
3	0.10
4	0.05
5	0.05

TABLE 15.2 PROBABILITY DISTRIBUTION FOR DEMAND

Demand (units per week)	Probability of Demand
10	0.10
13	0.20
18	0.40
23	0.20
26	0.10



Demand Probability Distribution

(Units Per Period)

Probabilty of Demand	Lower Range Probability	Demand (Units)
0.10	0.00	10
0.20	0.10	13
0.40	0.30	18
0.20	0.70	23
0.10	0.90	26

Lead Time Probability Distribution

Probability of Lead Time	Lower Range	Protection Interval
0.35	0.00	1
0.45	0.35	2
0.10	0.80	3
0.05	0.90	4
0.05	0.95	5

Demand During Protection Interval Distribution

BINS	Demand	Frequency	Cumulative Percentage
10	**10**	24	**0.05**
22	**16**	84	**0.22**
34	**28**	141	**0.50**
46	**40**	145	**0.79**
58	**52**	49	**0.89**

Random Numbers

	Demand Per Period in Protection Interval									
Lead Time	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8	Period 9	Pe
0.1637	0.0833	0.5102	0.2518	0.5184	0.2025	0.7836	0.1628	0.5987	0.3116	0
0.6769	0.8404	0.6872	0.0135	0.1993	0.6971	0.3746	0.4923	0.4031	0.1828	0.
0.6847	0.7367	0.9619	0.9626	0.5671	0.5760	0.6335	0.1021	0.6465	0.1341	0.
0.2129	0.6666	0.7249	0.4595	0.2043	0.6652	0.6160	0.9479	0.7499	0.5985	0.
0.2541	0.0365	0.4230	0.2839	0.7738	0.3001	0.8734	0.0667	0.8477	0.7052	0
0.5016	0.6766	0.6239	0.3281	0.6489	0.4644	0.6653	0.5601	0.9461	0.5983	0
0.2	[illegible]	[illegible]	[illegible]	0.7195	0.0047	0.6908	0.5194	0.4332	0.1357	0.
0.2	[illegible]	[illegible]	[illegible]	0.3818	0.7743	0.9017	0.2498	0.8027	0.1885	0.
0.1	[illegible]	[illegible]	[illegible]	0.2327	0.6862	0.2880	0.2900	0.1826	0.0995	0
0.5	[illegible]	[illegible]	[illegible]	0.9474	0.1487	0.0656	0.8645	0.4554	0.1548	0
0.3	[illegible]	[illegible]	[illegible]	0.6635	0.1279	0.5530	0.1295	0.0743	0.8928	0.
0.8	[illegible]	[illegible]	[illegible]	0.7150	0.6383	0.4391	0.5368	0.7860	0.0648	0.
0.7255	0.8283	0.0944	0.8689	0.2304	0.4721	0.5777	0.0694	0.0299	0.4083	0
0.0536	0.3992	0.1281	0.8438	0.1844	0.7781	0.6994	0.8040	0.5647	0.2609	0
0.7607	0.6912	0.8048	0.3671	0.8267	0.8732	0.2523	0.0433	0.2588	0.4854	0.
0.2107	0.7238	0.9769	0.4767	0.4132	0.0630	0.7979	0.6091	0.5206	0.8320	0
0.2335	0.4533	0.6874	0.9631	0.6808	0.8151	0.3555	0.5553	0.7274	0.4522	0
0.0386	0.3003	0.4065	0.3505	0.9062	0.5987	0.4494	0.8158	0.4851	0.2432	0.
0.5019	0.9661	0.2541	0.6851	0.2483	0.0343	0.4933	0.2734	0.1725	0.7055	0.
0.4108	0.8023	0.0978	0.0657	0.9296	0.6807	0.9976	0.2785	0.5228	0.2677	0.
0.3744	0.5385	0.7282	0.7645	0.8855	0.0943	0.6941	0.2202	0.0919	0.0393	0
0.2122	0.7615	0.7820	0.0261	0.1679	0.0716	0.2459	0.2291	0.6456	0.6446	0
0.0925	0.8525	0.2320	0.0427	0.7376	0.6809	0.4502	0.7044	0.5545	0.8276	0.
0.8343	0.6395	0.7648	0.8997	0.3425	0.3840	0.6041	0.8482	0.3168	0.8569	0.

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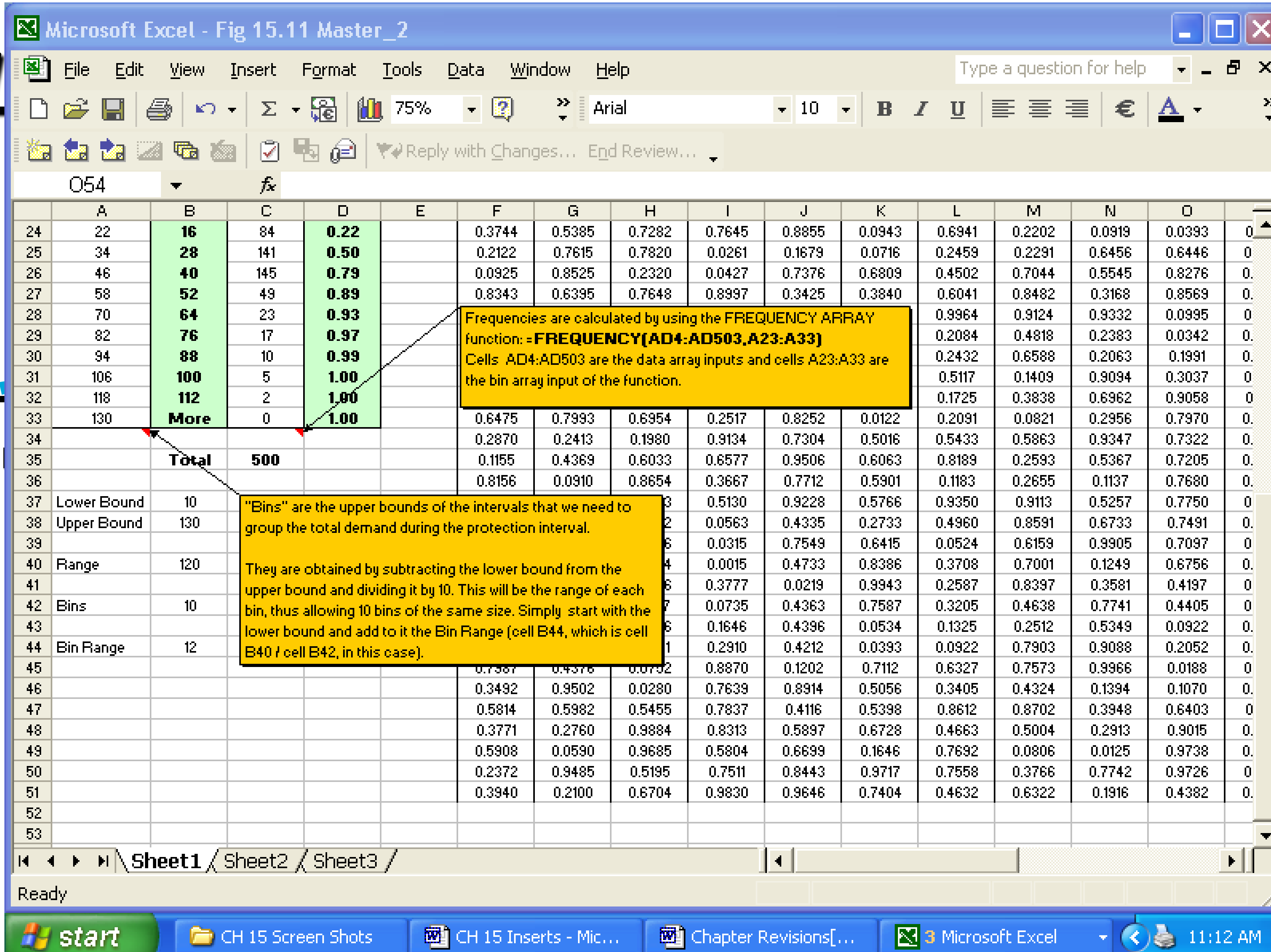
Demand for week 3

Figure 15.11(a)



Microsoft Excel - Fig 15.11 Master_2

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	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
24	22	**16**	84	**0.22**		0.3744	0.5385	0.7282	0.7645	0.8855	0.0943	0.6941	0.2202	0.0919	0.0393
25	34	**28**	141	**0.50**		0.2122	0.7615	0.7820	0.0261	0.1679	0.0716	0.2459	0.2291	0.6456	0.6446
26	46	**40**	145	**0.79**		0.0925	0.8525	0.2320	0.0427	0.7376	0.6809	0.4502	0.7044	0.5545	0.8276
27	58	**52**	49	**0.89**		0.8343	0.6395	0.7648	0.8997	0.3425	0.3840	0.6041	0.8482	0.3168	0.8569
28	70	**64**	23	**0.93**								0.9964	0.9124	0.9332	0.0995
29	82	**76**	17	**0.97**								0.2084	0.4818	0.2383	0.0342
30	94	**88**	10	**0.99**								0.2432	0.6588	0.2063	0.1991
31	106	**100**	5	**1.00**								0.5117	0.1409	0.9094	0.3037
32	118	**112**	2	**1.00**								0.1725	0.3838	0.6962	0.9058
33	130	**More**	0	**1.00**		0.6475	0.7993	0.6954	0.2517	0.8252	0.0122	0.2091	0.0821	0.2956	0.7970
34						0.2870	0.2413	0.1980	0.9134	0.7304	0.5016	0.5433	0.5863	0.9347	0.7322
35		**Total**	**500**			0.1155	0.4369	0.6033	0.6577	0.9506	0.6063	0.8189	0.2593	0.5367	0.7205
36						0.8156	0.0910	0.8654	0.3667	0.7712	0.5901	0.1183	0.2655	0.1137	0.7680
37	Lower Bound	10							0.5130	0.9228	0.5766	0.9350	0.9113	0.5257	0.7750
38	Upper Bound	130							0.0563	0.4335	0.2733	0.4960	0.8591	0.6733	0.7491
39									0.0315	0.7549	0.6415	0.0524	0.6159	0.9905	0.7097
40	Range	120							0.0015	0.4733	0.8386	0.3708	0.7001	0.1249	0.6756
41									0.3777	0.0219	0.9943	0.2587	0.8397	0.3581	0.4197
42	Bins	10							0.0735	0.4363	0.7587	0.3205	0.4638	0.7741	0.4405
43									0.1646	0.4396	0.0534	0.1325	0.2512	0.5349	0.0922
44	Bin Range	12							0.2910	0.4212	0.0393	0.0922	0.7903	0.9088	0.2052
45									0.8870	0.1202	0.7112	0.6327	0.7573	0.9966	0.0188
46						0.3492	0.9502	0.0280	0.7639	0.8914	0.5056	0.3405	0.4324	0.1394	0.1070
47						0.5814	0.5982	0.5455	0.7837	0.4116	0.5398	0.8612	0.8702	0.3948	0.6403
48						0.3771	0.2760	0.9884	0.8313	0.5897	0.6728	0.4663	0.5004	0.2913	0.9015
49						0.5908	0.0590	0.9685	0.5804	0.6699	0.1646	0.7692	0.0806	0.0125	0.9738
50						0.2372	0.9485	0.5195	0.7511	0.8443	0.9717	0.7558	0.3766	0.7742	0.9726
51						0.3940	0.2100	0.6704	0.9830	0.9646	0.7404	0.4632	0.6322	0.1916	0.4382
52															
53															

Frequencies are calculated by using the FREQUENCY ARRAY function: =**FREQUENCY(AD4:AD503,A23:A33)**
Cells AD4:AD503 are the data array inputs and cells A23:A33 are the bin array input of the function.

"Bins" are the upper bounds of the intervals that we need to group the total demand during the protection interval.

They are obtained by subtracting the lower bound from the upper bound and dividing it by 10. This will be the range of each bin, thus allowing 10 bins of the same size. Simply start with the lower bound and add to it the Bin Range (cell B44, which is cell B40 / cell B42, in this case).

Sheet1 / Sheet2 / Sheet3

Ready

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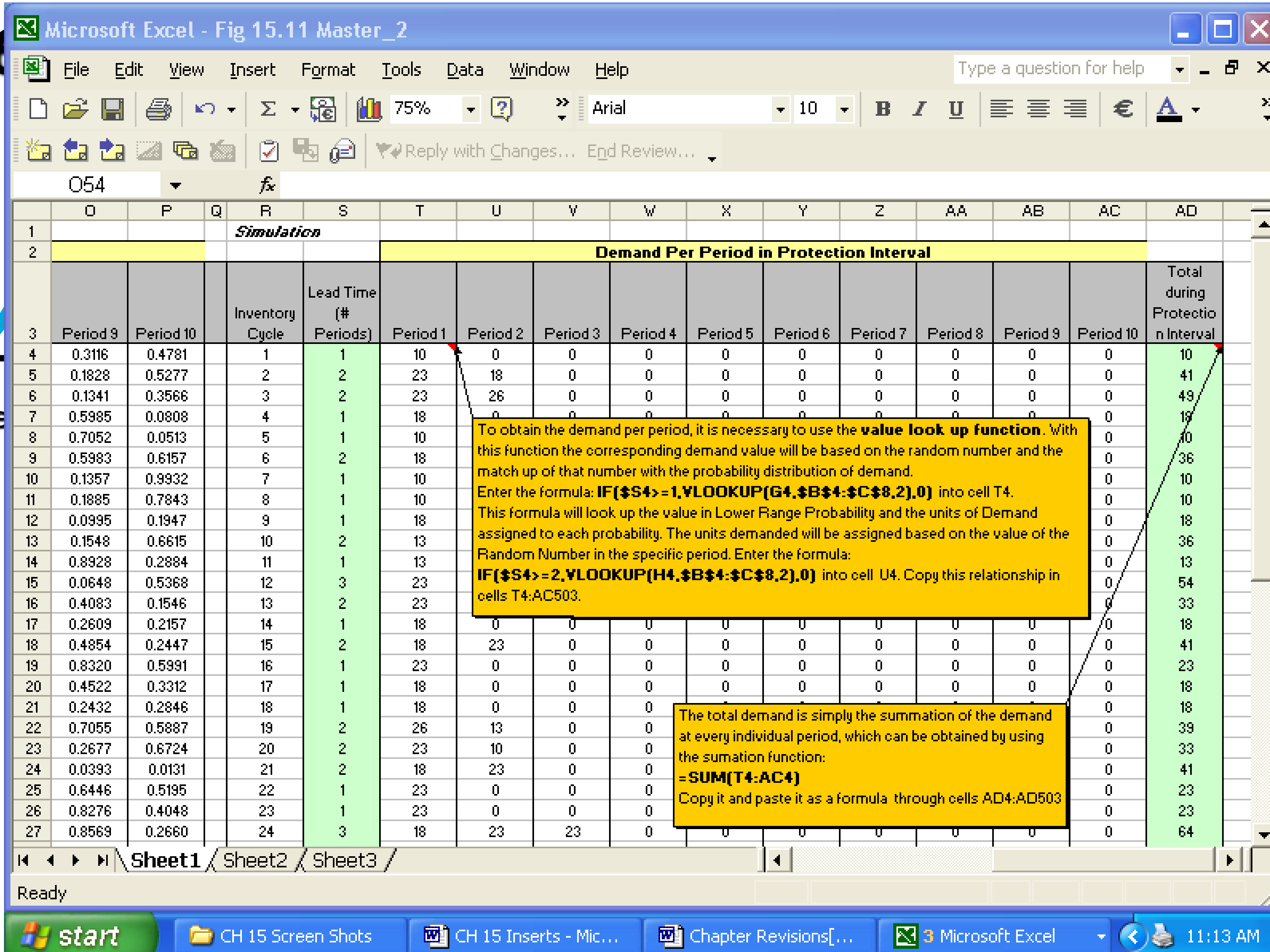
Demand for week 3

Figure 15.11(b)



	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD
3	Period 9	Period 10		Inventory Cycle	Lead Time (# Periods)	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8	Period 9	Period 10	Total during Protection Interval
4	0.3116	0.4781		1	1	10	0	0	0	0	0	0	0	0	0	10
5	0.1828	0.5277		2	2	23	18	0	0	0	0	0	0	0	0	41
6	0.1341	0.3566		3	2	23	26	0	0	0	0	0	0	0	0	49
7	0.5985	0.0808		4	1	18									0	18
8	0.7052	0.0513		5	1	10									0	10
9	0.5983	0.6157		6	2	18									0	36
10	0.1357	0.9932		7	1	10									0	10
11	0.1885	0.7843		8	1	10									0	10
12	0.0995	0.1947		9	1	18									0	18
13	0.1548	0.6615		10	2	13									0	36
14	0.8928	0.2884		11	1	13									0	13
15	0.0648	0.5368		12	3	23									0	54
16	0.4083	0.1546		13	2	23									0	33
17	0.2609	0.2157		14	1	18	0	0	0	0	0	0	0	0	0	18
18	0.4854	0.2447		15	2	18	23	0	0	0	0	0	0	0	0	41
19	0.8320	0.5991		16	1	23	0	0	0	0	0	0	0	0	0	23
20	0.4522	0.3312		17	1	18	0	0	0	0	0	0	0	0	0	18
21	0.2432	0.2846		18	1	18	0	0	0						0	18
22	0.7055	0.5887		19	2	26	13	0	0						0	39
23	0.2677	0.6724		20	2	23	10	0	0						0	33
24	0.0393	0.0131		21	2	18	23	0	0						0	41
25	0.6446	0.5195		22	1	23	0	0	0						0	23
26	0.8276	0.4048		23	1	23	0	0	0						0	23
27	0.8569	0.2660		24	3	18	23	23	0	0	0	0	0	0	0	64

Figure 15.11(c)

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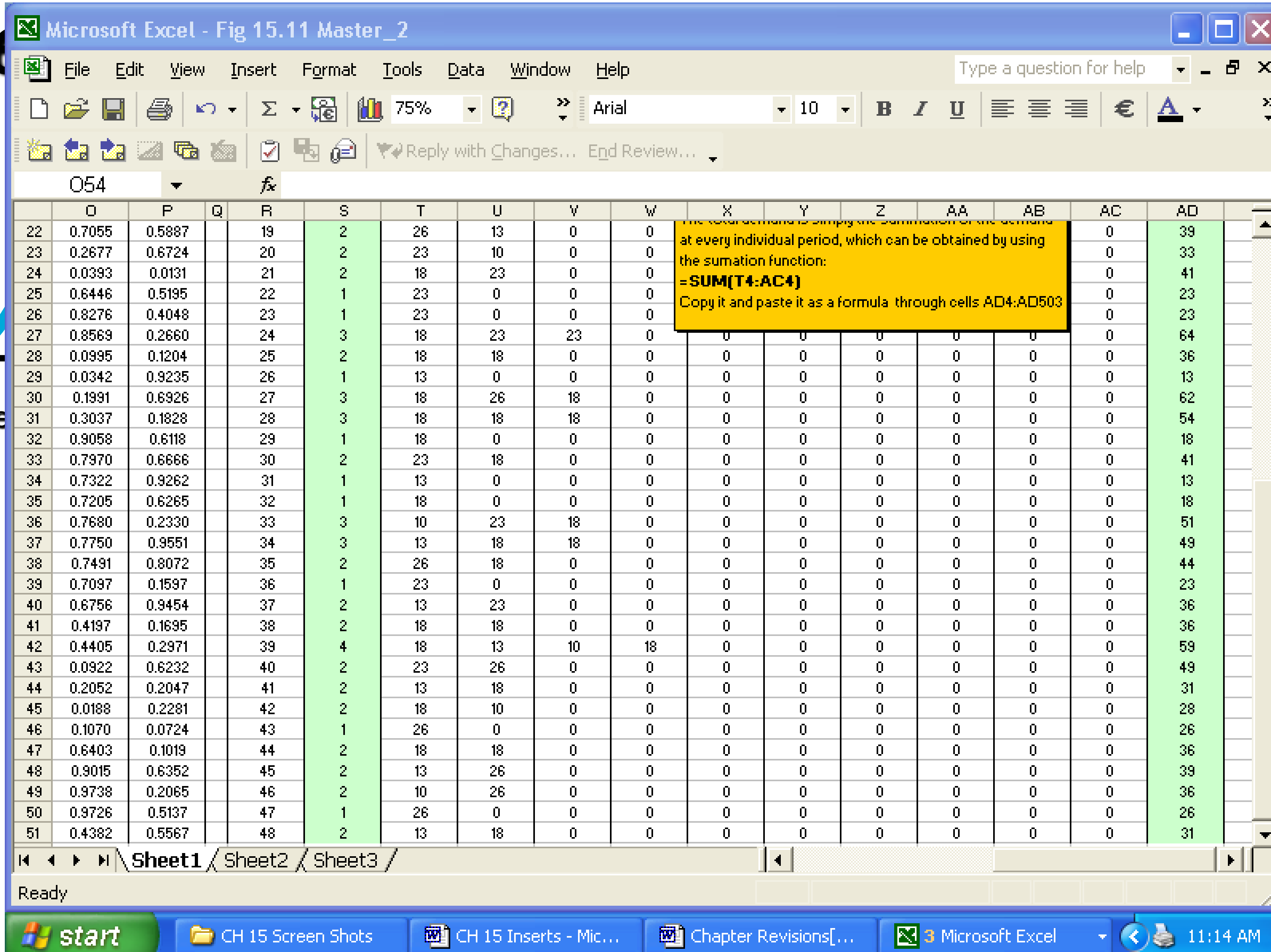
Demand for week 3



	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD
22	0.7055	0.5887		19	2	26	13	0	0						0	39
23	0.2677	0.6724		20	2	23	10	0	0						0	33
24	0.0393	0.0131		21	2	18	23	0	0						0	41
25	0.6446	0.5195		22	1	23	0	0	0						0	23
26	0.8276	0.4048		23	1	23	0	0	0						0	23
27	0.8569	0.2660		24	3	18	23	23	0	0	0	0	0	0	0	64
28	0.0995	0.1204		25	2	18	18	0	0	0	0	0	0	0	0	36
29	0.0342	0.9235		26	1	13	0	0	0	0	0	0	0	0	0	13
30	0.1991	0.6926		27	3	18	26	18	0	0	0	0	0	0	0	62
31	0.3037	0.1828		28	3	18	18	18	0	0	0	0	0	0	0	54
32	0.9058	0.6118		29	1	18	0	0	0	0	0	0	0	0	0	18
33	0.7970	0.6666		30	2	23	18	0	0	0	0	0	0	0	0	41
34	0.7322	0.9262		31	1	13	0	0	0	0	0	0	0	0	0	13
35	0.7205	0.6265		32	1	18	0	0	0	0	0	0	0	0	0	18
36	0.7680	0.2330		33	3	10	23	18	0	0	0	0	0	0	0	51
37	0.7750	0.9551		34	3	13	18	18	0	0	0	0	0	0	0	49
38	0.7491	0.8072		35	2	26	18	0	0	0	0	0	0	0	0	44
39	0.7097	0.1597		36	1	23	0	0	0	0	0	0	0	0	0	23
40	0.6756	0.9454		37	2	13	23	0	0	0	0	0	0	0	0	36
41	0.4197	0.1695		38	2	18	18	0	0	0	0	0	0	0	0	36
42	0.4405	0.2971		39	4	18	13	10	18	0	0	0	0	0	0	59
43	0.0922	0.6232		40	2	23	26	0	0	0	0	0	0	0	0	49
44	0.2052	0.2047		41	2	13	18	0	0	0	0	0	0	0	0	31
45	0.0188	0.2281		42	2	18	10	0	0	0	0	0	0	0	0	28
46	0.1070	0.0724		43	1	26	0	0	0	0	0	0	0	0	0	26
47	0.6403	0.1019		44	2	18	18	0	0	0	0	0	0	0	0	36
48	0.9015	0.6352		45	2	13	26	0	0	0	0	0	0	0	0	39
49	0.9738	0.2065		46	2	10	26	0	0	0	0	0	0	0	0	36
50	0.9726	0.5137		47	1	26	0	0	0	0	0	0	0	0	0	26
51	0.4382	0.5567		48	2	13	18	0	0	0	0	0	0	0	0	31

Figure 15.11(d)

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Demand for week 3

Other Probabilistic Models (1 of 4)

- When data on demand during lead time are not available, there are other models available
    1. When demand is variable and lead time is constant
    2. When lead time is variable and demand is constant
    3. When both demand and lead time are variable

Other Probabilistic Models (2 of 4)

Demand is variable and lead time is constant

$$\text{ROP} = (\textit{Average}\ \text{daily demand} \times \text{Lead time in days}) + Z\sigma_{\text{dLT}}$$

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$$\sigma_{dLT} = \sigma_d \sqrt{\text{Lead time}}$$

$$\sigma_d = \text{Standard deviation of demand per day}$$

Probabilistic Example (2 of 3)

Average daily demand (normally distributed) = 15

Lead time in days (constant) = 2

Standard deviation of daily demand = 5

Service level = 90%

Z for 90% = 1.28

From Appendix I

$$\text{ROP} = (15 \text{ units} \times 2 \text{ days}) + Z\sigma_{dLT}$$
$$= 30 + 1.28(5)\left(\sqrt{2}\right)$$
$$= 30 + 9.02 = 39.02 \approx 39$$

Safety stock is about 9 computers

Gauss table and random distribution Appendix

	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Other Probabilistic Models (3 of 4)

Lead time is variable and demand is constant

$$\text{ROP} = (\text{Daily demand} \times \textit{Average}\ \text{lead time in days}) + Z \times (\text{Daily demand}) \times \sigma_{LT}$$

where $\sigma_{LT} = \text{Standard deviation of lead time in days}$

Probabilistic Example (3 of 3)

Daily demand (constant) = 10

Average lead time = 6 days

Standard deviation of lead time = σ_{LT} = 1

Service level = 98%, so Z (from Appendix I) = 2.055

$$\text{ROP} = (10 \text{ units} \times 6 \text{ days}) + 2.055(10 \text{ units})(1)$$
$$= 60 + 20.55 = 80.55$$

Reorder point is about 81 cameras

Other Probabilistic Models (4 of 4)

Both demand and lead time are variable

$$\text{ROP} = (\text{Average daily demand} \times \text{Average lead time}) + Z\sigma_{LT}$$

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e

$$\sigma_d = \text{Standard deviation of demand per day}$$

$$\sigma_{LT} = \text{Standard deviation of lead time in days}$$

$$\sigma_{dLT} = \sqrt{(\text{Average lead time} \times \sigma_{d^2}) + \left(\text{Average daily demand}\right)^2 \sigma^2{}_{LT}}$$

Probabilistic Example

Average daily demand (normally distributed) = 150

Standard deviation = σ_d = 16

Average lead time 5 days (normally distributed)

Standard deviation = σ_{LT} = 1 day

Service level = 95%, so Z = 1.645 (from Appendix I)

$$\text{ROP} = (150 \text{ packs} \times 5 \text{ days}) + 1.645\sigma_{dLT}$$

$$\sigma_{dLT} = \sqrt{\left(5 \text{ days} \times 16^2\right) + \left(150^2 \times 1^2\right)} = \sqrt{\left(5 \times 256\right) + \left(22,500 \times 1\right)}$$

$$= \sqrt{\left(1,280\right) + \left(22,500\right)} = \sqrt{23,780} \cong 154$$

$$\text{ROP} = (150 \times 5) + 1.645(154) \cong 750 + 253 = 1,003 \text{ packs}$$

Single-Period Model

- Only *one* order is placed for a product
- Units have little or no value at the end of the sales period

$$C_s = \text{Cost of shortage} = \text{Sales price/unit} - \text{Cost/unit}$$

$$C_o = \text{Cost of overage} = \text{Cost/unit} - \text{Salvage value}$$

$$\text{Service level} = \frac{C_s}{C_s + C_o}$$

Single-Period Example (1 of 2)

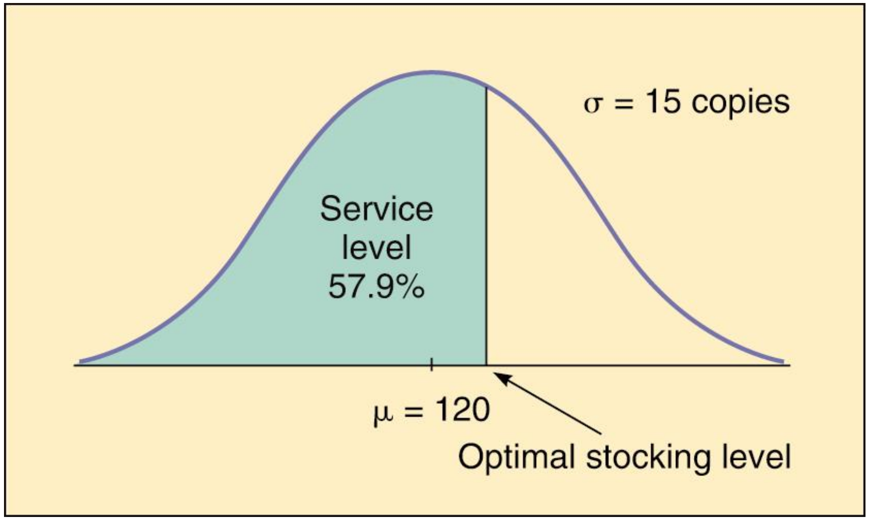
Average demand = μ = 120 papers/day

Standard deviation = σ = 15 papers

C_s = cost of shortage = \$1.25 − \$.70 = \$.55

C_o = cost of overage = \$.70 − \$.30 = \$.40

$$\text{Service level} = \frac{C_s}{C_s + C_o} = \frac{.55}{.55 + .40} = \frac{.55}{.95} = .579$$



Single-Period Example (2 of 2)

From Appendix I, for the area .579, $Z \cong .199$

The optimal stocking level

= 120 copies + (.199)(σ)

= 120 + (.199)(15) = 120 + 3 = 123 papers

The stockout risk = 1 − Service level

= 1 − .579 = .421 = 42.1%

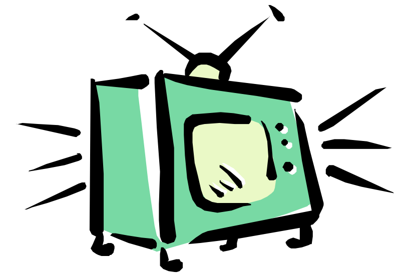
OTHER QUESTION

- The demand at Arnold Palmer Hospital for a specialized surgery pack is 60 per week, virtually every week. The lead time from McKesson, its main supplier, is normally distributed, with a mean of 6 weeks for this product and a standard deviation of 2 weeks. A 90% weekly service level is desired. Find the ROP.

Safety stock

- The inclusion of safety stock (ss) changed the expression to
  - ROP = d * L + ss
  - The amount of safety stock maintained depends on the cost of incurring a stockout and the cost of holding the extra inventory. Annual stockout cost is computed as follows:
- Annual stockout costs =
  - The sum of the units short for each demand level * The probability of that demand level * The stockout cost / unit * The number of orders per year

Periodic Review Systems



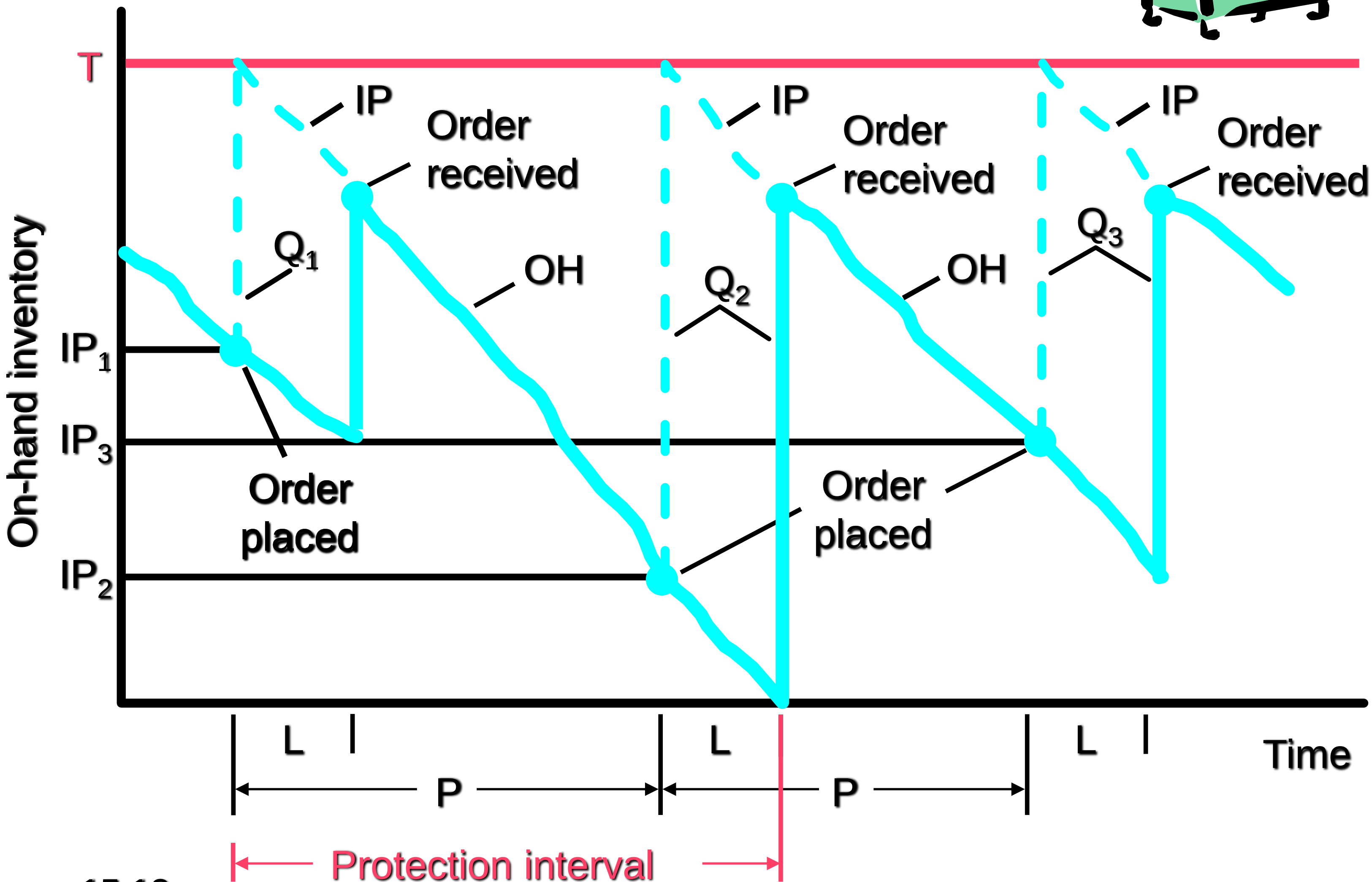
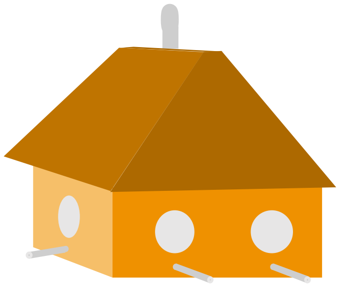


Figure 15.12

Example
Calculating P and T

- Again, let us return to the bird feeder example. Recall that demand for the bird feeder is normally distributed with a mean of 18 units per week and a standard deviation in weekly demand of 5 units. The lead time is 2 weeks, and the business operates 52 weeks per year. The Q system developed in Example 15.6 called for an EOQ of 75 units and a safety stock of 9 units for a cycle-service level of 90 percent. What is the equivalent *P system? What is the total cost ? Answers are to be rounded to the nearest integer.*

Periodic Review Systems



T

Bird feeder— Calculating *P* and *T*

σ_t = 5 units L = 2 weeks   cycle/service level = 90%

EOQ = 75 units D = (18 units/week)(52 weeks) = 936 units

$$P = \frac{\text{EOQ}}{D}(52) = \frac{75}{936}(52) = 4.2 \text{ or } 4 \text{ weeks}$$

$$\sigma_{P+L} = \sigma_t\sqrt{P+L} = 5\sqrt{6} = 12 \text{ units}$$

T = Average demand during the protection interval + Safety stock

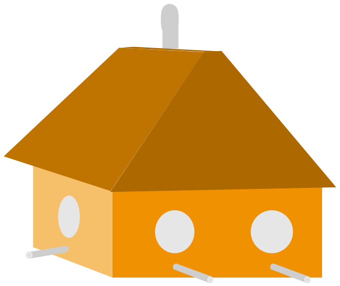
$= d\,(P + L) + z\sigma_{P+L}$

= (18 units/week)(6 weeks) + 1.28(12 units) = 123 units

Protection interval

Example 15.8

Periodic Review Systems



On-hand inventory

Bird feeder— Calculating *P* and *T*

σ_t = 18 units L = 2 weeks   cycle/service level = 90%

EOQ = 75 units D = (18 units/week)(52 weeks) = 936 units

P = 4 weeks T = 123 units

$$C = \frac{4(18)}{2}(\$15) + \frac{936}{4(18)}(\$45) + 15(\$15)$$

$$C = \$540 + \$585 + \$225 = \$1350$$

Protection interval

Example 15.8

Comparison of Q and P Systems

P Systems

- Convenient to administer
- Orders may be combined
- IP only required at review

Q Systems

- Individual review frequencies
- Possible quantity discounts
- Lower, less-expensive safety stocks


QUIZ
INVENTORIES

1-ABC analysis divides on-hand inventory into three classes, based on:

a) unit price.
b) the number of units on hand.
c) annual demand.
d) annual dollar values.

2-Cycle counting:

a) provides a measure of inventory turnover.
b) assumes that all inventory records must be verified with the same frequency.
c) is a process by which inventory records are periodically verified.
d) all of the above.

3-The two most important inventory-based questions answered by the typical inventory model are:

a) when to place an order and the cost of the order.
b) when to place an order and how much of an item to order.
c) how much of an item to order and the cost of the order.
d) how much of an item to order and with whom the order should be placed.

4- Extra units in inventory to help reduce stockouts are called:

a) reorder point.
b) safety stock.
c) just-in-time inventory.
d) all of the above

5-The difference(s) between the basic EOQ model and the production order quantity model is(are) that:

a) the production order quantity model does not require the assumption of known, constant demand.
b) the EOQ model does not require the assumption of negligible lead time.
c) the production order quantity model does not require the assumption of instantaneous delivery.
d) all of the above.

6-The EOQ model with quantity discounts attempts to determine:

a) the lowest amount of inventory necessary to satisfy a certain service level.
b) the lowest purchase price.
c) whether to use a fixed-quantity or fixed-period order policy.
d) how many units should be ordered.
e) the shortest lead time.

7-The appropriate level of safety stock is typically determined by:

a) minimizing an expected stockout cost.
b) choosing the level of safety stock that assures a given service level.
c) carrying sufficient safety stock so as to eliminate all stockouts.
d) annual demand

Problem 1

Booker's Book Bindery divides inventory items into three classes, according to their dollar usage. Calculate the usage values of the following inventory items and determine which is most likely to be classified as an A item.

PART NUMBER	DESCRIPTION	QUANTITY USED PER YEAR	UNIT VALUE ($)
1	Boxes	500	3.00
2	Cardboard (square feet)	18,000	0.02
3	Cover stock	10,000	0.75
4	Glue (gallons)	75	40.00
5	Inside covers	20,000	0.05
6	Reinforcing tape (meters)	3,000	0.15
7	Signatures	150,000	0.45

Problem 2

A regional warehouse purchases hand tools from various suppliers and then distributes them on demand to retailers in the region. The warehouse operates five days per week, 52 weeks per year. Only when it is open can orders be received. The following data are estimated for 3/8-inch hand drills with double insulation and variable speeds:

Average daily demand = 100 drills
Standard deviation of daily demand (σ_t) = 30 drills
Lead time (L) = 3 days
Holding cost (H) = \$9.40/unit/year
Ordering cost (S) = \$35/order
Cycle-service level = 92 percent

The warehouse uses a continuous review (Q) system.

a. What order quantity, Q and reorder point, R, should be used?
b. If on-hand inventory is 40 units, there is one open order for 440 drills, and there are no backorders, should a new order be placed?

	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Problem 3

Suppose that a periodic review (P) system is used at the warehouse, but otherwise the data are the same as in Solved Problem 5.

a. Calculate the P (in workdays, rounded to the nearest day) that gives approximately the same number of orders per year as the EOQ.
b. What is the value of the target inventory level, T? Compare the P system to the Q system in Solved Problem 5.
c. It is time to review the item. On-hand inventory is 40 drills; there is a scheduled receipt of 440 drills and no backorders. How much should be reordered?

David Rivera Optical has determined that its reorder point for eyeglass frames is 50 ($d \times L$) units. Its carrying cost per frame per year is $5, and stockout (or lost sale) cost is $40 per frame. The store has experienced the following probability distribution for inventory demand during the lead time (reorder period). The optimum number of orders per year is six.

NUMBER OF UNITS	PROBABILITY
30	.2
40	.2
ROP → 50	.3
60	.2
70	.1
	1.0

How much safety stock should David Rivera keep on hand?

APPROACH ▶ The objective is to find the amount of safety stock that minimizes the sum of the additional inventory holding costs and stockout costs. The annual holding cost is simply the holding cost per unit multiplied by the units added to the ROP. For example, a safety stock of 20 frames, which implies that the new ROP, with safety stock, is 70 (= 50 + 20), raises the annual carrying cost by $5(20) = $100.

However, computing annual stockout cost is more interesting. For any level of safety stock, stockout cost is the expected cost of stocking out. We can compute it, as in Equation (12-12), by multiplying the number of frames short (Demand – ROP) by the probability of demand at that level, by the stockout cost, by the number of times per year the stockout can occur (which in our case is the number of orders per year). Then we add stockout costs for each possible stockout level for a given ROP.[4]

SOLUTION ▶ We begin by looking at zero safety stock. For this safety stock, a shortage of 10 frames will occur if demand is 60, and a shortage of 20 frames will occur if the demand is 70. Thus the stockout costs for zero safety stock are:

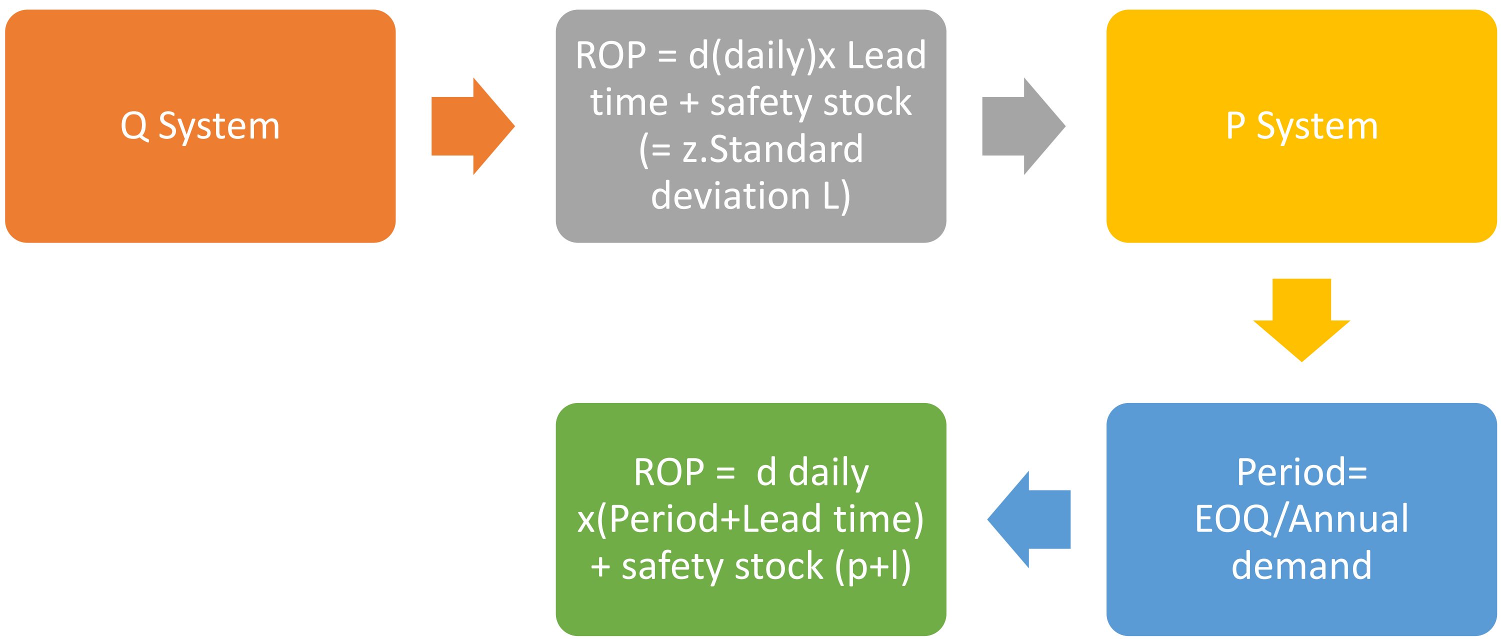
(10 frames short)(.2)($40 per stockout)(6 possible stockouts per year)
+ (20 frames short)(.1)($40)(6) = $960

The following table summarizes the total costs for each of the three alternatives:

SAFETY STOCK	ADDITIONAL HOLDING COST	STOCKOUT COST	TOTAL COST
20	(20)($5) = $100	$ 0	$100
10	(10)($5) = $ 50	(10) (.1) ($40) (6) = $240	$290
0	$ 0	(10) (.2) ($40) (6) + (20) (.1) ($40) (6) = $960	$960

The safety stock with the lowest total cost is 20 frames. Therefore, this safety stock changes the reorder point to 50 + 20 = 70 frames.

INSIGHT ▶ The optical company now knows that a safety stock of 20 frames will be the most economical decision.


Q System
ROP = d(daily)x Lead time + safety stock (= z.Standard deviation L)
P System
Period= EOQ/Annual demand
ROP = d daily x(Period+Lead time) + safety stock (p+l)