# $\Omega \cong \diamond\mathcal{W}$

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## 1 Definition

### 1.1 $\Omega$

There is a correct process $c$ such that **eventually** $\Omega_p = c$ **forever** for every correct process $p$.

### 1.2 $\diamond\mathcal{W}$

Eventual Weak Accuracy + Weak Completeness :

**Eventual weak accuracy:** *There is a time after which some* correct process is never suspected by any correct process.

**Weak Completeness:** Every faulty process is eventually permanently suspected by *some* correct process.

## 2 System Assumptions

1. Complete network topology.
2. Asynchronous identified processes: a process and its identifier are used equivalently.
3. Asynchronous reliable links (not necessarily FIFO).

## 3 Notations

- $V$: the set of processes
- $Correct \subseteq V$: the set of correct processes.
- $Faulty \subseteq V$: the set of faulty processes.
- $X_p$: the value of variable $X$ of process $p$.
- $X_p^t$: the value of variable $X$ of process $p$ at time $t$.

## 4 $T_{\Omega\to\diamond\mathcal{W}}$

**Algorithm 1** $T_{\Omega\to\diamond\mathcal{W}}$, code for every process $p$

```
Function T_{Ω→◇W}(p)
    return V \ {Ω_p}
End Function
```

**Question 1.** Prove that $T_{\Omega \to \diamond \mathcal{W}}$ satisfies weak completeness.

**Question 2.** Prove that $T_{\Omega \to \diamond \mathcal{W}}$ satisfies eventual weak accuracy.

## 5 $T_{\diamond \mathcal{W} \to \Omega}$

**Question 1.** Following the principles presented in the lesson, propose an algorithm $T_{\diamond \mathcal{W} \to \Omega}$.

*Notations:* Please use the following variables:

- $Leader \in V$, initialized to $p$.
- $C[]$, array of integers indexed on $V$, every cell is initialized to 0.

**Question 2.** Show the following lemma.

**Lemma 1.** *$\forall p \in Correct$, $\forall q \in Faulty$, $\forall t \in \mathbb{N}$, $\exists t' > t$ such that $C[q]_p^t < C[q]_p^{t'}$.*

**Question 3.** Show the following lemma.

**Lemma 2.** *$\exists p \in Correct$ such that $\exists k, t \in \mathbb{N}$ such that $\forall q \in Correct$, $\forall t' \geq t$, $C[p]_q^{t'} \leq k$.*

**Question 4.** Show the following lemma.

**Lemma 3.** $\exists t \in \mathbb{N}$ *such that* $\forall p \in Correct$, $\forall t' \geq t$, $C_p^t[Leader_p^t] = C_p^{t'}[Leader_p^{t'}]$.

**Question 5.** Show the following corollary.

**Corollary 1.** $\exists t \in \mathbb{N}$ *such that* $\forall p \in Correct$, $\forall t' \geq t$, $Leader_p^t = Leader_p^{t'}$.

**Question 6.** Show the following lemma.

**Lemma 4.** $\exists \ell \in Correct$ *and* $\exists t \in \mathbb{N}$ *such that* $\forall t' \geq t$, $\forall p \in Correct$, $Leader_p^{t'} = \ell$.