

$$\Omega \cong \diamond\mathcal{W}$$

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1 Definition

1.1 Ω

There is a correct process c such that **eventually** $\Omega_p = c$ **forever** for every correct process p .

1.2 $\diamond\mathcal{W}$

Eventual Weak Accuracy + Weak Completeness :

Eventual weak accuracy: *There is a time after which some correct process is never suspected by any correct process.*

Weak Completeness: Every faulty process is eventually permanently suspected by *some* correct process.

2 System Assumptions

1. Complete network topology.
2. Asynchronous identified processes: a process and its identifier are used equivalently.
3. Asynchronous reliable links (not necessarily FIFO).

3 Notations

- V : the set of processes
- $Correct \subseteq V$: the set of correct processes.
- $Faulty \subseteq V$: the set of faulty processes.
- X_p : the value of variable X of process p .
- X_p^t : the value of variable X of process p at time t .

4 $T_{\Omega \rightarrow \diamond\mathcal{W}}$

Algorithm 1 $T_{\Omega \rightarrow \diamond\mathcal{W}}$, code for every process p

- 1: **Function** $T_{\Omega \rightarrow \diamond\mathcal{W}}(p)$
 - 2: **return** $V \setminus \{\Omega_p\}$
 - 3: **End Function**
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Question 1. Prove that $T_{\Omega \rightarrow \diamond \mathcal{W}}$ satisfies weak completeness.

Question 2. Prove that $T_{\Omega \rightarrow \diamond \mathcal{W}}$ satisfies eventual weak accuracy.

5 $T_{\diamond \mathcal{W} \rightarrow \Omega}$

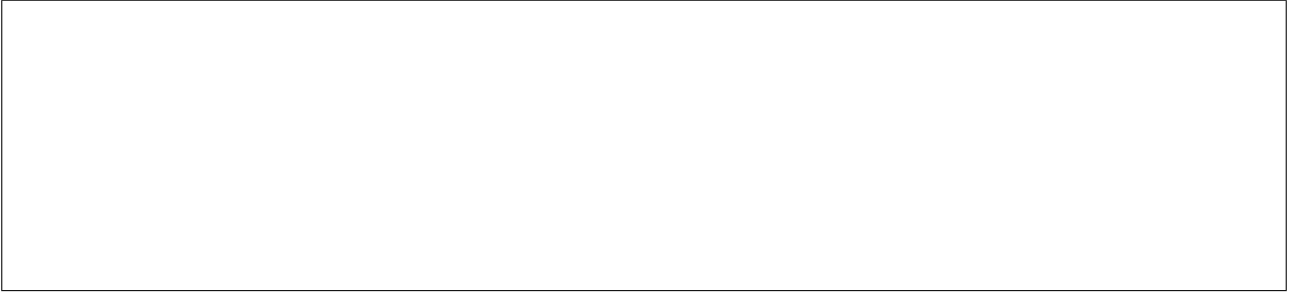
Question 1. Following the principles presented in the lesson, propose an algorithm $T_{\diamond \mathcal{W} \rightarrow \Omega}$.

Notations: Please use the following variables:

- *Leader* $\in V$, initialized to p .
- $C[]$, array of integers indexed on V , every cell is initialized to 0.

Question 2. Show the following lemma.

Lemma 1. $\forall p \in \text{Correct}, \forall q \in \text{Faulty}, \forall t \in \mathbb{N}, \exists t' > t$ such that $C[q]_p^t < C[q]_p^{t'}$.



Question 3. Show the following lemma.

Lemma 2. $\exists p \in \text{Correct}$ such that $\exists k, t \in \mathbb{N}$ such that $\forall q \in \text{Correct}, \forall t' \geq t, C[p]_q^{t'} \leq k$.



Question 4. Show the following lemma.

Lemma 3. $\exists t \in \mathbb{N}$ such that $\forall p \in \text{Correct}, \forall t' \geq t, C_p^t[\text{Leader}_p^t] = C_p^{t'}[\text{Leader}_p^{t'}]$.

Question 5. Show the following corollary.

Corollary 1. $\exists t \in \mathbb{N}$ such that $\forall p \in \text{Correct}, \forall t' \geq t, \text{Leader}_p^t = \text{Leader}_p^{t'}$.

Question 6. Show the following lemma.

Lemma 4. $\exists \ell \in \text{Correct}$ and $\exists t \in \mathbb{N}$ such that $\forall t' \geq t, \forall p \in \text{Correct}, \text{Leader}_p^{t'} = \ell$.